

Concrete elastic modulus effects on cracked flexural stiffness reduction factor of RC beams

Nguyen Hoang Tung¹, Sok Heang^{1,2*}, Khuc Thien Thanh^{1,2}

¹ Faculty of Civil Engineering, Van Lang School of Technology, Van Lang University

² Institute of Postgraduate Education, Van Lang University

KEYWORDS

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Stiffness reduction factor
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ABSTRACT

This paper presents an analytical model for evaluating the cracked flexural stiffness (EI_{cr}) and stiffness reduction factor (n) of reinforced concrete beams, in which the concrete elastic modulus (E_c) is treated as a variable input parameter. The reduction of stiffness is critical and the current research was not been comprehensively clarified. n is defined as the ratio of the nonlinear stiffness at tensile steel yielding to the elastic stiffness of the section. The model is formulated based on sectional force equilibrium and strain compatibility conditions, allowing the determination of the moment–curvature relationship ($M-\varphi$) and EI_{cr} . The reliability of the proposed analytical approach is validated through comparison with the Kent–Park concrete model for beams with different configurations. The analytical predictions are in good agreement with the essential trends of the moment–curvature response, with an average error of 4.24% in predicting the n . Based on the validated model, a parametric study is conducted to investigate the influence of the E_c on n . Concrete compressive strengths ranging from (30-150MPa) are considered, together with 10 coarse aggregate types and three mineral admixture conditions. The analysis is performed on four representative beam configurations, with each beam evaluated under 750 calculation cases. The results indicate that the E_c has a significant effect on the n , even when the cross-sectional geometry and reinforcement ratio remain unchanged. Moreover, n derived from design-standard E_c formulas may lead to noticeable differences in EI_{cr} evaluation, particularly in the high-strength concrete.

1. Introduction

In RC frame structures subjected to lateral loads, beam flexural deformation plays a critical role in the global deformation mechanism. Numerous studies have shown that frame shear deformation, mainly contributed by beam flexural deformation, accounts for more than half of the total lateral displacement, whereas column bending and shear deformation often play secondary roles [1, 2]. Consequently, inaccurate modeling of beam flexural stiffness may lead to substantial errors in evaluating lateral stiffness, natural periods, and internal force distribution within the structural system. The cracked flexural stiffness of RC beams is commonly expressed through the product EI_{cr} , where I_{cr} is the cracked moment of inertia and E_c is the concrete elastic modulus. Although nonlinear dynamic analysis is capable of fully capturing the structural response, it requires sophisticated modeling and advanced expertise in nonlinear structural dynamics [3]. Therefore, over the past decades, most research has focused on sectional analysis approaches to establish moment–curvature relationships or develop empirical stiffness expressions [4-6], with particular emphasis on the cracked moment of inertia I_{cr} . The classical expression proposed by Branson [4] has been widely used to determine cracked flexural stiffness, although it is mainly applicable to serviceability conditions. Khuntia and Ghosh [5]

developed a method based on moment–curvature relationships, allowing a more detailed description of stiffness degradation with increasing deformation and providing simplified expressions for RC design. More recent studies, such as Godínez-Domínguez et al. [6], conducted large-scale parametric analyses to investigate the effects of reinforcement ratio, concrete strength, and sectional geometry on cracked stiffness. These results indicate that cracked flexural stiffness is highly variable and dependent on multiple parameters. In design practice, many current standards recommend adopting constant stiffness reduction factors for RC beams in linear elastic models. These values are typically expressed as fractions of the gross moment of inertia I_g [7-13]. Although this approach is convenient, it inherently assumes that cracked flexural stiffness remains constant and independent of specific material characteristics. Many studies have shown that using fixed stiffness reduction factors may lead to inaccurate stiffness estimation, particularly under dynamic and seismic loading conditions [5, 6, 14-18]. Besides the cracked moment of inertia, the concrete elastic modulus E_c is the other fundamental parameter directly governing flexural stiffness through EI_g . Nevertheless, in many previous studies and design practices, the influence of E_c has often been treated as secondary or implicitly assumed through simplified relationships with compressive strength [16]. In reality, the concrete elastic modulus may

*Corresponding author: heang.2485802010009@vanlanguni.vn

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vary significantly depending on aggregate type, mineral admixtures, density, curing conditions, and evaluation methods [16, 19]. In particular, for high-strength concrete, E_c strongly depends on the nature of coarse aggregates and may differ considerably even for concretes with the same compressive strength [20-22]. Takafumi Noguchi et al. [19] pointed out that standard-based elastic modulus formulas do not adequately capture the effects of aggregate and mineral admixtures, especially for high-strength concrete. Tena-Colunga et al. [16] further emphasized that in the stiffness matrix of RC beam elements, E_c plays an equally important role as the sectional moment of inertia, and neglecting its variability may lead to significant errors in stiffness evaluation. Although some studies have addressed the influence of E_c , the results remain scattered and have not been systematically linked to the stiffness reduction factor of RC beams. Therefore, despite extensive research on cracked flexural stiffness, most existing works primarily focus on sectional geometry and reinforcement ratio effects through I_{cr} , while the isolated influence of the concrete elastic modulus on cracked stiffness and stiffness reduction factors has not yet been comprehensively clarified. In particular, the relationship between elastic modulus determined using aggregate- and admixture-dependent models and those prescribed by different design standards remains insufficiently explored.

This paper investigates the cracked flexural stiffness of reinforced concrete beams through stiffness reduction factors, in which the concrete elastic modulus is determined using both current design-standard expressions and formulations accounting for variability due to compressive strength, aggregate type, and admixture conditions. Based on these considerations, an analytical model satisfying sectional equilibrium and strain compatibility is developed to establish the moment–curvature relationship and determine stiffness reduction factors. The analytical model is validated through comparison with the Kent–Park concrete material model [23] for beams with different configurations, a model widely recognized in RC structural analysis and design [24]. After validation, elastic modulus values derived from design standards and from aggregate-dependent formulations are employed to assess stiffness reduction factors. The obtained results clarify the extent to which elastic modulus determination affects the evaluation of the cracked flexural stiffness, thereby improving consistency in RC stiffness modeling.

The remainder of the paper is organized as follows. Section 2 presents elastic modulus expressions from current design standards and from aggregate- and admixture-dependent formulations, together with comparative analysis results. Section 3 describes the theoretical background and computational procedure for constructing moment–curvature relationships and determining stiffness reduction factors. Section 4 discusses the validation results against the Kent–Park model and presents the parametric study outcomes. Finally, Section 5 summarizes the main conclusions and the design-oriented formulas for predicting the cracked stiffness reduction factor are presented in the Appendix.

2. Concrete Elastic Modulus

2.1. Standard-Based Formulations for the Concrete Elastic Modulus

The concrete elastic modulus is commonly estimated using empirical expressions proposed in design standards and recommendations, which are primarily based on its relationship with the concrete compressive strength. According to ACI 318 [7], the elastic modulus is estimated as $E_c = 4700\sqrt{f_c}$. In Eurocode 2 [25], the mean elastic modulus is given by $E_{cm} = 22000(f_{cm}/10)^{0.3}$. The CSA A23.3 [9] proposes the expression $E_c = (3300\sqrt{f_c} + 6900)(\gamma_c/2300)^{1.5}$, where γ_c is the concrete density. Similarly, NZS 3101 [10] adopts $E_c = (3320\sqrt{f_c} + 6900)(\gamma_c/2300)^{1.5}$. In the above equations, E_c and E_{cm} denote the elastic modulus of concrete (MPa), while f_c and $f_{cm} = 8 + f_c$ represent the characteristic and mean compressive strengths of concrete (MPa), respectively and γ_c is the density of concrete.

2.2. Aggregate and Mineral Admixture Effects on the Concrete Elastic Modulus

In order to more comprehensively describe the variability of the concrete elastic modulus under different material conditions, Takafumi Noguchi et al. [19] proposed a generalized expression for estimating the concrete elastic modulus E_c . This formulation simultaneously accounts for the effects of compressive strength, concrete density, coarse aggregate type, and mineral admixtures through a series of correction factors. The expression is given as follows $E_c = 3.35 \times 10^4 k_1 k_2 (\gamma_c/2400)^2 (f_c/60)^{1/3}$ where γ_c is the density of concrete, f_c is the concrete compressive strength, and k_1 and k_2 are two correction factors, k_1 and k_2 , are introduced to incorporate the effects of coarse aggregate and mineral admixture variations. The correction factors k_1 for different aggregate types and k_2 for various mineral admixtures are presented in Table 2 and 3.

Table 1 compiles the material cases used to determine the concrete elastic modulus in this research, consisting of combinations of 10 coarse aggregate types and three mineral admixture conditions in Table 2.

In this study, the concrete elastic modulus is determined based on the expressions prescribed in current design standards, including ACI 318, Eurocode 2, CSA A23.3, and NZS 3101, together with the formulation proposed by Takafumi Noguchi et al. In order to ensure comparability, the same range of concrete compressive strength values is adopted for all models, varying from 30 MPa to 150 MPa. This range represents commonly encountered concrete strength levels, from normal-strength to high-strength concrete. The concrete density is assumed to remain constant and is taken as 2400 kg/m³, which is consistent with typical normal-weight concrete used in reinforced concrete structures. For the Noguchi et al. model, the aggregate- and admixture-related correction factors are selected based on the

representative aggregate and mineral admixture types presented in Tables 2 and 3.

Table 1. Coarse aggregates adapted from [19].

Aggregate type	k_1
River gravel	1,005
Crushed graywacke	1,002
Crushed quartzitic aggregate	0,931
Crushed limestone	1,207
Crushed andesite	0,902
Crushed basalt	0,922
Crushed clayslate	0,928
Crushed cobblestone	0,955
Blast-furnace slag	0,987
Calcined bauxite	1,163

Table 2. Mineral admixtures adapted from [19].

Type of mineral admixtures	k_2
Silica fume, ground-granulated blast-furnace slag, fly ash fume	0,95
Fly ash	1,10
Another mineral admixture	1,00

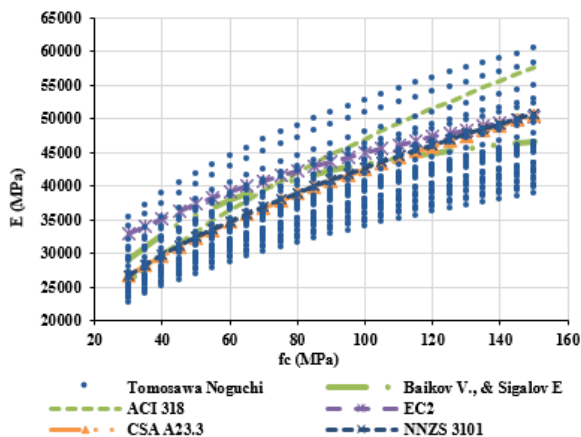


Figure 1. Variations of the concrete elastic modulus with concrete compressive strength for different aggregate and admixture conditions Takafumi Noguchi et al. [19] and comparison to standard-based formulations [7, 9, 10, 25].

Figure 1 illustrates the relationship between the concrete elastic modulus and the corresponding compressive strength, as determined by various expressions adopted in current design standards and by the model proposed by Takafumi Noguchi et al. It can be observed that although all formulations exhibit the general trend of increasing elastic modulus with increasing concrete compressive strength, the absolute values of E_c differ significantly among the models, particularly in the

medium- to high-strength concrete range. In the low- to moderate-strength region, the expressions provided in ACI 318, Eurostandard 2, and CSA A23.3 yield relatively similar results, reflecting the assumption that the elastic modulus is primarily governed by the compressive strength of concrete. However, as the concrete strength increases, the discrepancies among the formulations become more pronounced. The design-standard expressions tend to predict higher elastic modulus values compared with Takafumi Noguchi et al. model, particularly for high-strength concrete. This observation is consistent with the remarks made by Noguchi et al., who pointed out that standard formulas, which are generally developed from limited datasets with restricted aggregate types, may overestimate the elastic modulus in certain cases. The model proposed by Takafumi Noguchi et al. indicates that the concrete elastic modulus is not solely dependent on compressive strength, but is also significantly influenced by concrete density, coarse aggregate type, and mineral admixtures through a series of correction factors. Consequently, for the same compressive strength level, this model may provide different elastic modulus values depending on specific material characteristics. This feature more realistically reflects practical situations where concrete is produced using diverse aggregate sources and admixture conditions, particularly under local material environments. The substantial variation in elastic modulus values shown in Figure 1 suggests that adopting different standard-based expressions may lead to considerable differences in the evaluation of reinforced concrete beam flexural stiffness.

3. Analytical Model

In this section, an analytical model based on the nonlinear material behavior is presented to establish the moment–curvature relationship of reinforced concrete beams, from which the cracked flexural stiffness and the corresponding stiffness reduction factor can be determined. The model is formulated by simultaneously satisfying the force equilibrium condition and the strain compatibility condition at the cross-section, in accordance with the fundamental assumptions of conventional reinforced concrete beam theory. The analytical model is formulated under the assumption that plane sections remain plane after bending. A perfect bond between concrete and reinforcing steel is also assumed, with no relative slip. Concrete is assumed to carry only compressive stresses; the tensile behavior of cracked concrete is neglected. The material behavior of both concrete and steel is described using nonlinear stress–strain relationships. In the ductile failure mechanism of the beam, it is assumed that concrete reaches its ultimate compressive strain limit before the tensile reinforcement fully yields. The influence of concrete confinement provided by transverse reinforcement on the flexural stiffness at the onset of longitudinal steel yielding is considered negligible [6].

Figure 2 shows the stress–strain relationships of concrete and reinforcing steel. A simplified bilinear stress–strain diagram, as presented in the TCVN 5574:2018 standard [26], is adopted to represent

the concrete behavior in compression. The relationship between the concrete compressive stress (σ_b) and the corresponding strain (ε_b) is expressed by the following equations:

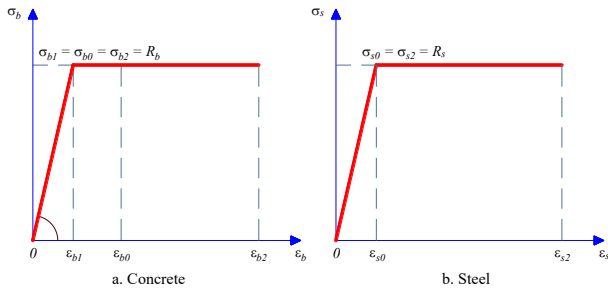


Figure 2. Stress–strain relationship for concrete adopted in the analytical model.

In Eqs. (1) and (2), these values are associated with the characteristic states of the simplified concrete stress–strain diagram depicted in Figure 1a.

$$\sigma_b = E_{b,red} \varepsilon_b \text{ when } 0 \leq \varepsilon_b \leq \varepsilon_{b1} \quad (1)$$

$$\sigma_b = R_b \text{ when } \varepsilon_{b1} \leq \varepsilon_b \leq \varepsilon_{b2} \quad (2)$$

For $\varepsilon_{b1} = R_b / E_{b,red}$ and variable concrete elastic modulus

$$E_{b,red} = R_b / \varepsilon_{b1,red}$$

Following [27, 28], $E_b = 55000 f_c / (27 + f_c)$ when $R_b = 0,8 f_c$, $R_{bt,ser} = 5 f_c / (45 + f_c)$

The relationship between tensile stress and tensile strain (Figure 2b) is defined by the following equations:

$$\sigma_s = \varepsilon_s E_s \text{ when } 0 \leq \varepsilon_s \leq \varepsilon_{s0} \quad (3)$$

$$\sigma_s = R_s \text{ when } \varepsilon_{s0} \leq \varepsilon_s \leq \varepsilon_{s2} \quad (4)$$

In Eq (4) $\varepsilon_{s2} = 0,025$, The compressive stress–strain behavior of reinforcing steel follows the same form as its tensile response. Accordingly, it can be described by Eqs. (3) and (4), with tensile terms replaced by the corresponding compressive quantities.

The moment–curvature relationship of reinforced concrete beams is established by gradually increasing the sectional curvature and determining the corresponding equilibrium state at each calculation

step. For a given curvature value, the strain distribution across the cross-section is defined according to the plane-section assumption. Based on this strain distribution, the stresses in both concrete and reinforcing steel are obtained through their respective stress–strain relationships. The axial force equilibrium condition on the cross-section is then used to determine the position of the neutral axis. Once the force equilibrium condition is satisfied, the corresponding bending moment is calculated by summing the moments of all internal forces with respect to the neutral axis. This procedure is repeated for progressively increasing curvature values until the cross-section reaches its ultimate limit state. From the resulting moment–curvature relationship, key characteristic points of the section, including the yield moment, ultimate moment, and their corresponding curvatures, are identified. The cracked flexural stiffness of the beam is subsequently derived from the slope of the moment–curvature curve at the stage when the tensile reinforcement just begins to yield plastically ($EI_{cr} = M_y / \varphi_y$). This stiffness serves as the basis for determining the stiffness reduction factor in subsequent analyses ($n = EI_{cr} / EI_g$).

4. Validation

4.1. Validation of Analytical Method

The beams listed in Table 3 were classified into four groups (B01–B04) to validate the analytical model under different sectional mechanisms. Group B01 investigates the influence of tensile reinforcement by systematically varying the tensile steel area and reinforcement ratio μ_1 , while keeping other parameters constant. Group B02 evaluates the effect of compressive reinforcement, where the compressive steel area and ratio μ_2 are varied while tensile reinforcement and beam geometry remain unchanged. Group B03 examines the combined influence of both tensile and compressive reinforcement by varying both reinforcement areas to generate different reinforcement combinations. Group B04 focuses on the effect of concrete compressive strength, with reinforcement layout and cross-sectional geometry kept constant.

Table 3. Geometric and reinforcement properties of the reinforced concrete beam groups considered in this study.

Group	Beam ID	b	h	h_0	a	a'	ϕ_1	A_s	μ_1	ϕ_2	A_{s2}	μ_2	Tỉ số
		mm	mm	mm	mm	mm	mm	mm ²	%	mm	mm ²	%	(μ_1 / μ_2)
B01	B01-1	250	500	460	40	39	3 ϕ 20	942,478	0,820	3 ϕ 18	763,41	0,66	0,81
	B01-2	250	500	459	41	39	3 ϕ 22	1,140,398	0,994	3 ϕ 18	763,41	0,67	0,669
	B01-3	250	500	457,5	42,5	39	3 ϕ 25	1,472,622	1,288	3 ϕ 18	763,41	0,67	0,518
	B01-4	250	500	456	44	39	3 ϕ 28		1,62	3 ϕ 18	763,41	0,67	0,413
	B01-5	250	500	455	45	39	3 ϕ 30	2,120,575	1,864	3 ϕ 18	763,41	0,67	0,36
B02	B02-1	300	550	505	45	39	3 ϕ 30	2,120,575	1,4	3 ϕ 18	763,41	0,5	0,36
	B02-2	300	550	505	45	40	3 ϕ 30	2,120,575	1,4	3 ϕ 20	942,48	0,62	0,444

Group	Beam ID	b	h	h_0	a	a'	ϕ_1	A_s	μ_1	ϕ_2	A_{s2}	μ_2	Tỉ số
		mm	mm	mm	mm	mm	mm	mm ²	%	mm	mm ²	%	(μ_1 / μ_2)
	B02-3	300	550	505	45	41	3 ϕ 30	2,120,575	1,4	3 ϕ 22	1140,4	0,75	0,538
	B02-4	300	550	505	45	42,5	3 ϕ 30	2,120,575	1,4	3 ϕ 25	1472,62	0,97	0,694
	B02-5	300	550	505	45	44	3 ϕ 30	2,120,575	1,4	3 ϕ 28	1847,26	1,22	0,871
B03	B03-1	300	650	588,2	61,8	40	3 ϕ 22 + 2 ϕ 22	1,900,664	1,077	3 ϕ 20	942,48	0,53	0,496
	B03-2	300	650	583	67	41	3 ϕ 22 + 3 ϕ 22	2,280,796	1,304	3 ϕ 22	1140,4	0,65	0,5
	B03-3	300	650	585,5	64,5	42,5	3 ϕ 25 + 2 ϕ 25	2,454,369	1,397	3 ϕ 25	1472,62	0,84	0,6
	B03-4	300	650	580	70	44	3 ϕ 25 + 3 ϕ 25	2,945,243	1,693	3 ϕ 28	1847,26	1,06	0,627
	B03-5	300	650	582,8	67,2	45	3 ϕ 28 + 2 ϕ 28	3,078,761	1,761	3 ϕ 30	2120,58	1,21	0,689
B04	B04-1	400	800	727	73	44	4 ϕ 28 + 4 ϕ 28	4,926,017	1,694	4 ϕ 28	2463,01	0,85	0,5
	B04-2	400	800	727	73	44	4 ϕ 28 + 4 ϕ 28	4,926,017	1,694	4 ϕ 28	2463,01	0,85	0,5
	B04-3	400	800	727	73	44	4 ϕ 28 + 4 ϕ 28	4,926,017	1,694	4 ϕ 28	2463,01	0,85	0,5
	B04-4	400	800	727	73	44	4 ϕ 28 + 4 ϕ 28	4,926,017	1,694	4 ϕ 28	2463,01	0,85	0,5
	B04-5	400	800	727	73	44	4 ϕ 28 + 4 ϕ 28	4,926,017	1,694	4 ϕ 28	2463,01	0,85	0,5

Note: b is the beam width, h is the total section depth, and h_0 is the effective depth. a and a' denote the concrete cover to tensile and compressive reinforcement, respectively. ϕ_1 , ϕ_2 is the bar diameter; A_s and A_{s2} are the areas of tensile and compressive reinforcement; μ_1 and μ_2 are the corresponding reinforcement ratios.

Table 4. Mechanical properties of concrete and reinforcing steel adopted in the analytical model.

Group	Beam ID	f_c	R_b	$R_{bt,ser}$	E_c	R_s	R_{sc}	E_s
		(MPa)	(MPa)	(MPa)	(MPa)	(MPa)	(MPa)	(MPa)
B01	B01-(1-4)	30	24	2	28947,368	400	400	200000
B02	B02-1-4	35	28	2,188	31048,387	400	400	200000
B03	B03-1-4	40	32	2,353	32835,821	400	400	200000
B04	B04-1	30	24	2	28947,368	400	400	200000
	B04-2	35	28	2,188	31048,387	400	400	200000
	B04-3	40	32	2,353	32835,821	400	400	200000
	B04-4	45	36	2,5	34375	400	400	200000
	B04-5	50	40	2,632	35714,286	400	400	200000

Note: f_c is the concrete compressive strength; R_b and $R_{bt,ser}$ denote the compressive and tensile strengths of concrete; E_c is the elastic modulus of concrete. R_s and R_{sc} are the yield strengths of tensile and compressive reinforcing steel, respectively, and E_s is the elastic modulus of reinforcing steel.

Figure 3-6 presents the moment–curvature relationship of reinforced concrete beams obtained from the analytical model employing a bilinear concrete stress–strain diagram and from the Kent–Park model. The two curves exhibit good agreement over the full response range of the section, from the initial elastic phase to the nonlinear post-cracking stage. The differences between the two models mainly appear in the large-strain region; however, the overall trend and curve shape remain similar. To quantitatively evaluate the level of agreement, moment values corresponding to the same curvature levels were compared. The results indicate a clear similarity between the analytical model predictions and those from the Kent–Park model, these findings demonstrate that the analytical method is capable of capturing

the essential flexural behavior of beams in the cracked stage. Table 5 presents a quantitative comparison between the analytical model proposed in this study and the Kent–Park concrete model for beams belonging to groups B01–B04. The compared parameters include the yield moment M_y , yield curvature ϕ_y , ultimate moment M_u , ultimate curvature ϕ_u , and the cracked stiffness reduction factor. For the yield moment M_y , the values predicted by the analytical model range from 156,056 kN·m to 1281,765 kN·m, whereas the corresponding Kent–Park results range from 156,197 kN·m to 1278,542 kN·m. For the yield curvature ϕ_y , the analytical model provides values between 0.00427 and 0.00793 (1/m), while the Kent–Park model yields results between

0,00404 and 0,00748 (1/m). The differences are small and consistent across beam groups, indicating the ability of the analytical model to properly describe the strain state at steel yielding. For the ultimate moment M_{u} , the analytical results range from 161,069 kN·m to 1358,053 kN·m, while the Kent–Park model predicts values from 161,073 kN·m to 1359,167 kN·m. These results demonstrate an excellent agreement between the two models in predicting the ultimate flexural capacity of the section. For the ultimate curvature φ_u , the analytical model provides values in the range of 0,0268–0,23036 (1/m), whereas the Kent–Park model yields values between 0,02679 and 0,2306 (1/m). Although ultimate curvature is highly sensitive to nonlinear material modeling, the deviations remain small and acceptable for structural analysis purposes. Regarding the cracked stiffness reduction factor, the analytical model predicts values ranging from 0,325 to 0,559, while the Kent–Park model produces results between 0,3386 and 0,5883. The corresponding ratios fall within 0,94–0,98, with an average value of approximately 0,96, indicating a high level of agreement between the two models in estimating the stiffness degradation of reinforced concrete beams after cracking.

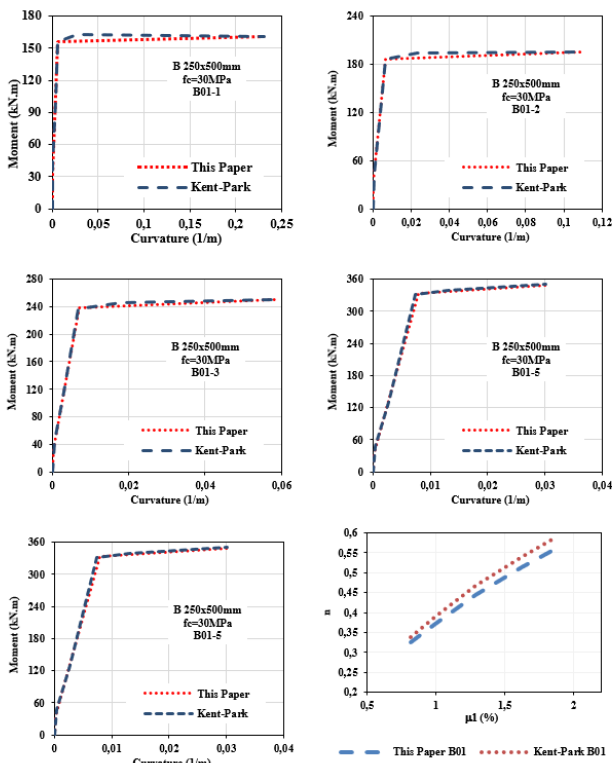


Figure 3. Variations of the moment–curvature response and corresponding stiffness reduction factors obtained from the proposed analytical model and the Kent–Park [23] concrete model for Group B01 beams.

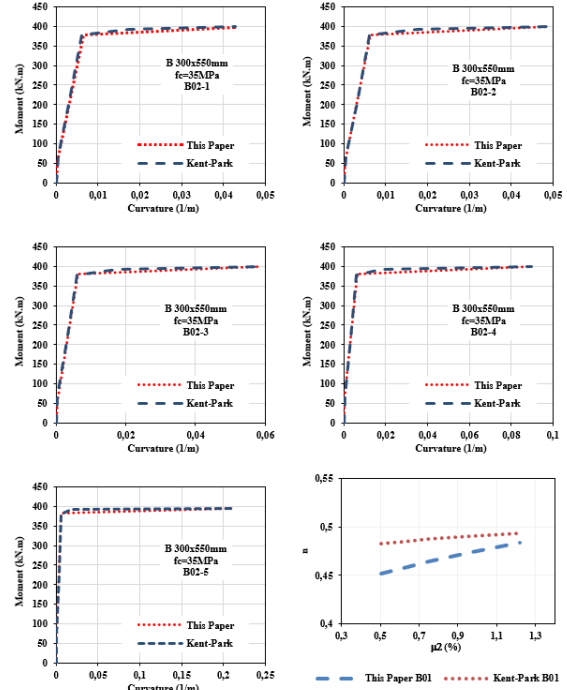


Figure 4. Variations of the moment–curvature response and corresponding stiffness reduction factors obtained from the proposed analytical model and the Kent–Park [27] concrete model for Group B02 beams.

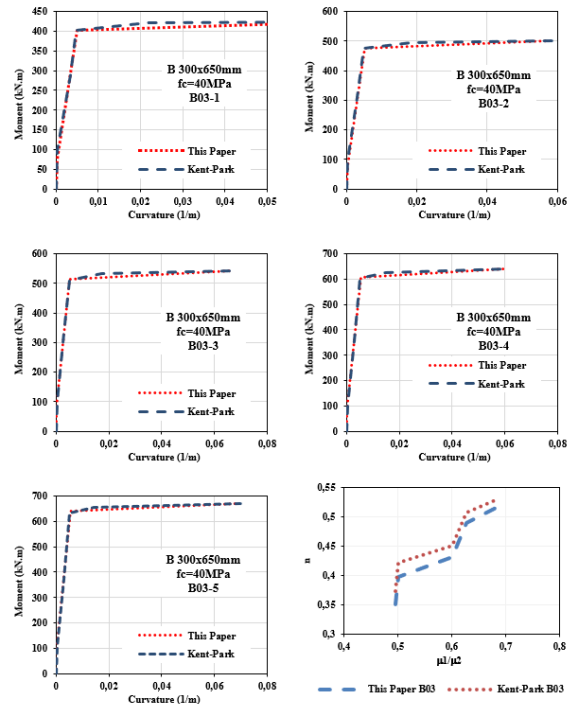


Figure 5. Variations of the moment–curvature response and corresponding stiffness reduction factors obtained from the proposed analytical model and the Kent–Park [27] concrete model for Group B03 beams.

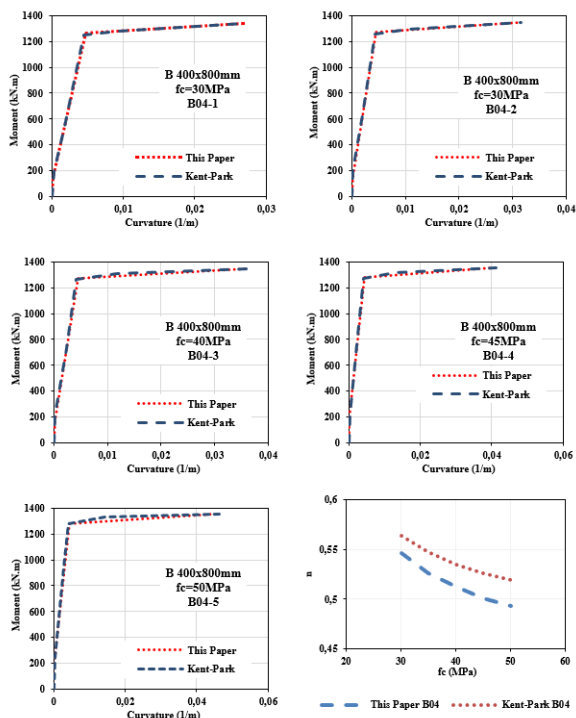


Figure 6. Variations of the moment–curvature response and corresponding stiffness reduction factors obtained from the proposed analytical model and the Kent–Park [27] concrete model for Group B04 beams.

4.2. Parametric Study

A systematic parametric investigation is presented to examine the influence of the concrete elastic modulus on the stiffness reduction factor of reinforced concrete beams. For each reinforced concrete beam, the concrete compressive strength was considered within the range of 30 MPa to 150 MPa, covering both normal-strength and high-strength concrete. This strength range was divided into 25 distinct strength levels, representing commonly used concrete grades in construction

practice. At each strength level, the corresponding concrete elastic modulus was determined for different types of coarse aggregates. In this study, a total of 10 aggregate types were considered in order to reflect the diversity of concrete material sources in practice. For each aggregate type, the elastic modulus was further examined under three different material conditions, including two cases with mineral admixtures and one case without mineral admixtures. These scenarios allow the combined effects of aggregate type and mineral admixture on the concrete elastic modulus to be evaluated. In total, four representative beams were analyzed, corresponding to the beam groups B01–B04. The cross-sectional geometry, reinforcement ratio, and steel properties were held constant to isolate the effect of the concrete elastic modulus on the stiffness reduction factor.

Figure 7 presents the results of the parametric analysis on the effect of concrete elastic modulus variability—arising from differences in aggregate type and admixture conditions—on the stiffness reduction factor of reinforced concrete beams. In these plots, the stiffness reduction factors obtained from the analytical model are directly compared with the recommended values provided in current design standards. The results indicate that the stiffness reduction factor predicted by the analytical model varies over a wide range as the concrete elastic modulus changes, even though the cross-sectional geometry and reinforcement ratio remain unchanged. For the same concrete strength grade, variations in aggregate type and admixture condition lead to different elastic modulus values, resulting in significant differences in the stiffness reduction factor. Comparison with standard-prescribed stiffness reduction factors shows that standard values typically fall within a narrow range and do not fully reflect the variability induced by differences in the concrete elastic modulus. In some cases involving aggregates associated with lower elastic modulus values, the stiffness reduction factor predicted by the analytical model is considerably smaller than the value assumed in design standards. Conversely, for aggregates yielding higher elastic modulus values, the predicted factor tends to approach or even exceed the standard recommendations.

Table 5. Quantitative comparison between the proposed analytical model and the Kent–Park concrete model.

Group	Beam ID	This Paper					Kent-Park					n_1 / n_2
		M_y (kN.m)	φ_y (1/m)	M_u (kN.m)	φ_u (1/m)	n_1	M_y (kN.m)	φ_y (1/m)	M_u (kN.m)	φ_u (1/m)	n_2	
B01	B01-1	156,056	0,00637	161,069	0,23036	0,325	156,197	0,00612	161,073	0,2306	0,3386	0,96
	B01-2	186,952	0,00666	195,526	0,10942	0,373	186,795	0,00636	195,604	0,10896	0,3896	0,96
	B01-3	237,851	0,00711	195,526	0,10942	0,444	237,018	0,00675	195,604	0,10896	0,466	0,95
	B01-4	294,025	0,00759	308,979	0,03806	0,514	292,179	0,00717	309,486	0,03782	0,5405	0,95
	B01-5	334,053	0,00793	348,868	0,03039	0,559	331,521	0,00748	349,624	0,03021	0,5883	0,95
B02	B02-1	377,08	0,00647	398,47	0,04255	0,452	377,528	0,00605	399,119	0,04293	0,4831	0,94
	B02-2	378,371	0,0064	399,731	0,04902	0,458	377,981	0,00603	400,216	0,04945	0,4851	0,94
	B02-3	379,594	0,00633	400,277	0,05892	0,464	378,428	0,00601	400,612	0,05946	0,4872	0,95

Group	Beam ID	This Paper					Kent-Park					n_1 / n_2
		M_y	φ_y	M_u	φ_u	n_1	M_y	φ_y	M_u	φ_u	n_2	
		(kN.m)	(1/m)	(kN.m)	(1/m)		(kN.m)	(1/m)	(kN.m)	(1/m)		
	B02-4	381,249	0,00623	399,224	0,08913	0,474	379,064	0,00599	399,367	0,08991	0,4904	0,97
	B02-5	382,636	0,00612	395,115	0,21129	0,484	379,647	0,00596	395,142	0,21281	0,4935	0,98
B03	B03-1	401,621	0,00508	424,268	0,06888	0,351	402,928	0,00479	424,545	0,07011	0,3731	0,94
	B03-2	474,898	0,0053	502,073	0,05788	0,398	474,961	0,005	502,486	0,05891	0,421	0,95
	B03-3	513,837	0,00529	541,548	0,06723	0,431	512,128	0,00504	541,84	0,06839	0,4511	0,96
	B03-4	608,121	0,00551	640,489	0,06011	0,49	603,555	0,00528	640,869	0,06122	0,5075	0,97
	B03-5	639,422	0,00548	671,71	0,06888	0,518	633,382	0,00528	671,989	0,07014	0,5321	0,97
B04	B04-1	1268,039	0,00471	1337,33	0,0268	0,546	1250,845	0,00449	1339,274	0,02679	0,5636	0,97
	B04-2	1271,588	0,00456	1344,731	0,03126	0,526	1258,623	0,00434	1346,377	0,03154	0,5471	0,96
	B04-3	1275,107	0,00445	1350,282	0,03573	0,512	1265,874	0,00422	1351,706	0,03637	0,5352	0,96
	B04-4	1278,511	0,00435	1354,599	0,0402	0,501	1272,509	0,00412	1355,851	0,04129	0,5263	0,95
	B04-5	1281,765	0,00427	1358,053	0,04466	0,493	1278,542	0,0040376	1359,167	0,0463	0,5195	0,95
Average error											0,96	

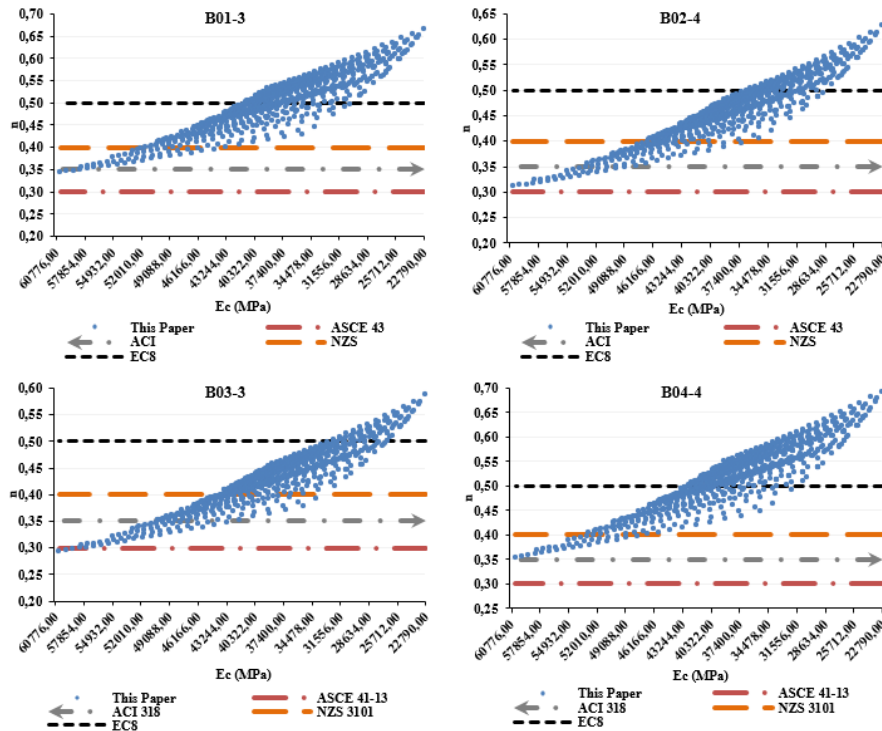


Figure 7. Variations of the stiffness reduction factor with E_c for different aggregate and admixture conditions, including comparison with standard-based stiffness reduction factors [7, 8, 10, 12] of B01-3, B02-4, B03-3, B04-4.

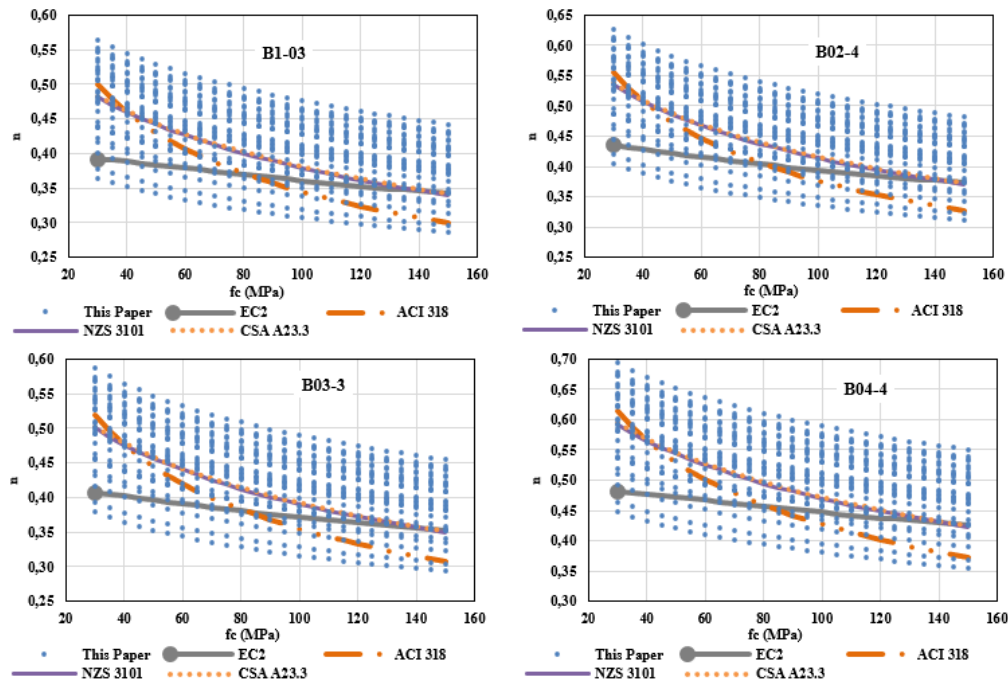


Figure 8. Variations of the stiffness reduction factor with the E_c for different aggregate and admixture conditions and comparison with elastic modulus formulations from different design standards [7, 9, 10, 25] of B01-3, B02-4, B03-3, B04-4.

Figure 8 illustrates the variation of the stiffness reduction factor of reinforced concrete beams with respect to the concrete compressive strength, in which the concrete elastic modulus values are determined using the formulas prescribed in different design standards. In these plots, the results obtained from the analytical model, where the concrete elastic modulus is treated as a variable parameter dependent on aggregate characteristics, are also presented for comparison. The curves in Figure 8 indicate that the stiffness reduction factor changes with concrete strength, exhibiting an overall decreasing trend as the compressive strength increases. However, the rate of reduction and the shape of the curves strongly depend on the elastic modulus formulation adopted. For the same concrete strength, different empirical expressions for estimating the concrete elastic modulus result in different stiffness reduction factor values. The discrepancies among the curves become more pronounced in the high-strength concrete range. In this domain, standard-based formulas provide significantly different elastic modulus values, which consequently lead to large differences in the derived stiffness reduction factors. In contrast, in the low- to medium-strength concrete range, the curves tend to converge, suggesting that the influence of the elastic modulus formulation is less significant. A comparison between the stiffness reduction factors inferred from standard-based elastic modulus expressions and those obtained from the formulation incorporating aggregate-dependent variability highlights substantial differences between the two approaches, particularly in the high-strength concrete region.

5. Conclusions

Based on the analyses and results obtained in this study, the following main conclusions can be drawn:

1. An analytical model was employed to describe the cracked flexural behavior and to determine the stiffness reduction factor of reinforced concrete beams. The validation results indicate that the proposed analytical model shows good agreement with the Kent–Park concrete model in reproducing the moment–curvature relationship, cracked flexural stiffness, and stiffness reduction factor, with a relatively low average deviation.

2. The parametric study reveals that the stiffness reduction factor of reinforced concrete beams is not a constant value but varies significantly with the concrete elastic modulus. For the same cross-sectional geometry and reinforcement ratio, variations in the concrete elastic modulus caused by different aggregate types and admixture conditions may lead to noticeable differences in the stiffness reduction factor, particularly in the high-strength concrete range.

3. The results also show that the stiffness reduction factors recommended in current design standards reflect only a small portion of the variability captured by the analytical model. In many cases, the standard-specified values fall outside the range obtained from the analytical analysis when the concrete elastic modulus is consistently considered according to material properties.

4. In addition, the use of different empirical expressions for determining the concrete elastic modulus in various design standards

results in different inferred stiffness reduction factors, even for the same concrete compressive strength. This discrepancy becomes more pronounced as the concrete strength increases, highlighting the sensitivity of the stiffness reduction factor to the method used for evaluating the elastic modulus.

5. Treating the concrete elastic modulus as a variable parameter allows a more rational evaluation of cracked flexural stiffness. The results confirm its key role in cracked behavior and highlight that considering its variability is essential for realistic stiffness reduction assessment.

Appendix

The design-oriented formulas below are intended to predict the stiffness reduction factor of RC beams at the stage when the tensile reinforcement just begins to yield are adopted from HEANG S and TÙNG N.H [18].

Case 1: when $0 \leq \varepsilon_b \leq \varepsilon_{b1}$ and $\varepsilon_s = \frac{R_s}{E_s}$

$$R_b A_s = \varepsilon_b E_{b,red} \cdot b \cdot \frac{1}{2} \cdot x_y + \sigma_{sc} A_{s2} \quad (5)$$

$$x_{yield} = \frac{2(R_s A_s - \sigma_{sc} A_{s2})}{\varepsilon_b E_{b,red} b} \quad (6)$$

$$\text{with } E_{b,red} = \frac{R_b}{\varepsilon_{b1,red}}, \quad \varepsilon_{b1,red} = 0.0015 \quad (7)$$

$$\sigma_{sc} = \varepsilon_{sc} \cdot E_s = \varepsilon_b \left(\frac{x_y - a'}{x_y} \right) \cdot E_s, \quad \varepsilon_b = \varepsilon_s \left(\frac{x_y}{h_0 - x_y} \right), \quad (8)$$

With ε_{sc} , Stress in the compression reinforcement

Depth of the compression zone

$$x_{yield} = \frac{-(2E_s \varepsilon_s A_{s2} + 2R_s A_s) + \sqrt{(2E_s \varepsilon_s A_{s2} + 2R_s A_s)^2 + \frac{8R_b b}{\varepsilon_{b1,red}} \cdot (A_s R_s \cdot h_0 + E_s \varepsilon_s A_{s2} \cdot a')}}{2R_b b \varepsilon_{b1,red}} \quad (9)$$

Yield moment

$$M_{yield} = \sigma_{sc} A_{s2} (h_0 - a') + \frac{\varepsilon_b R_b \cdot b}{\varepsilon_{b1,red}} \cdot \frac{x_y}{2} \left(h_0 - \frac{x_y}{3} \right) \quad (10)$$

Case 2: when $\varepsilon_{b1} \leq \varepsilon_b \leq \varepsilon_{b2}$

$$\frac{x_1}{x_{yield}} = \frac{\varepsilon_{b1}}{\varepsilon_b}, \quad x_1 = \frac{\varepsilon_{b1}}{\varepsilon_b} x_{yield}, \quad x_2 = \left(1 - \frac{\varepsilon_{b1}}{\varepsilon_b} \right) x_{yield} \quad (11)$$

$$R_b b x_2 + R_b b \frac{x_1}{2} = R_s A_s - \sigma_{sc} A_{s2} \quad (12)$$

Depth of the compression zone

Let

$$A = R_b b h_0 \left(1 + \frac{\varepsilon_{b1}}{\varepsilon_s} \right) + R_s A_s + E_s \varepsilon_s A_{s2} \quad (13)$$

$$B = \sqrt{\left(R_b b h_0 \left(1 + \frac{\varepsilon_{b1}}{\varepsilon_s} \right) + R_s A_s + E_s \varepsilon_s A_{s2} \right)^2 - 4R_b b \left(1 + \frac{\varepsilon_{b1}}{2\varepsilon_s} \right) \left(R_b b \frac{\varepsilon_{b1}}{2\varepsilon_s} h_0^2 + R_s A_s h_0 + E_s \varepsilon_s A_{s2} a' \right)} \quad (14)$$

$$C = 2R_b b \left(1 + \frac{\varepsilon_{b1}}{2\varepsilon_s} \right) \quad (15)$$

$$x_{yield} = \frac{A - B}{C} \quad (16)$$

Yield moment

$$M_{yield} = R_b b x_2 \left(h_0 - \frac{x_2}{2} \right) + R_b b \frac{x_1}{2} \left(h_0 - x_2 - \frac{x_1}{3} \right) + \sigma_{sc} A_{s2} (h_0 - a') \quad (17)$$

In this formula, x_1 and x_2 are the compression zone depths corresponding to the elastic and nonlinear strains of concrete, respectively.

Yield curvature

$$\varphi_{yield} = \frac{\varepsilon_b}{x_{yield}} \quad (18)$$

M_y : Yield moment, φ_y : Yield curvature

The cracked reduction factor of an RC beam can be determined:

$$n_{cracked} = \frac{EI_{cracked}}{EI_g}, \quad I_g = \frac{bh^3}{12}, \quad EI_{cracked} = \frac{M_{yield}}{\varphi_{yield}} \quad (19)$$

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