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Effect of geometric imperfections on stability and optimal design of shallow space structures

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KEYWORDS

Geometric imperfections
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Structural stability
Nonlinear analysis
Buckling behavior
Structural optimization
Imperfection sensitivity

ABSTRACT

Shallow space structures such as domes and roof trusses are widely used in large-span engineering due to their high structural efficiency. However, these systems are highly sensitive to geometric imperfections, which can significantly reduce their load-carrying capacity and trigger snap-through instability. In current engineering practice, structural design is often based on linear analysis or nonlinear analysis assuming perfect geometry, which may lead to unsafe and non-conservative results. This study investigates the influence of initial geometric imperfections on the stability and optimal design of shallow space structures. A geometrically nonlinear analysis is performed using a mixed finite element formulation capable of accurately tracing the equilibrium path. A 24-bar star dome is adopted as a benchmark structure, and three design scenarios are considered: linear analysis, nonlinear analysis with perfect geometry, and nonlinear analysis including geometric imperfections. The results reveal that linear and perfect nonlinear designs yield identical optimal solutions, masking critical stability issues. When moderate imperfections are considered, the optimal structural weight increases by approximately 17% and can rise dramatically for larger imperfections. These findings highlight the necessity of incorporating geometric imperfections in structural analysis and design to ensure safety and reliability.

1. Introduction

Shallow space structures, such as reticulated domes and roof trusses, are widely used in large-span engineering applications due to their high stiffness-to-weight ratio and structural efficiency. However, unlike deep structures, shallow systems are governed by pronounced geometric nonlinearity and are particularly susceptible to snap-through buckling under external loading [1,2]. This type of instability is characterized by a sudden transition between equilibrium states, often occurring at load levels significantly lower than the material strength.

A well-established feature of shallow structures is their high sensitivity to initial geometric imperfections. Small deviations arising from fabrication tolerances or assembly errors can lead to a substantial reduction in critical load-carrying capacity [3,4]. Classical studies in stability theory have demonstrated that imperfection-sensitive structures may exhibit a drastic degradation in stiffness and stability even for minor geometric disturbances [5]. As a result, neglecting such imperfections in analysis may lead to unsafe and non-conservative designs.

Despite this, current engineering practice often relies on linear analysis or geometrically nonlinear analysis assuming perfect geometry, primarily due to computational simplicity [6]. In the context of structural optimization, this issue becomes even more critical. Designs optimized under ideal conditions tend to eliminate structural redundancy, thereby increasing their vulnerability to imperfections and instability [7]. Consequently, there is a growing need to incorporate geometric imperfections directly into both analysis and design procedures.

To address these challenges, this study investigates the influence of geometric imperfections on the stability and optimal design of shallow space structures. A geometrically nonlinear analysis is performed using a mixed finite element formulation, which enables accurate tracing of the equilibrium path even near limit points. A 24-bar star dome is adopted as a benchmark structure, and three design scenarios are considered: linear analysis, nonlinear analysis with perfect geometry, and nonlinear analysis including geometric imperfections. The results aim to quantify the impact of imperfections on structural performance and provide insights for safer and more reliable design.

2. Theoretical formulation

2.1. Geometrically nonlinear formulation

The mechanical behavior of shallow space structures is strongly influenced by geometric nonlinearity due to large displacements. For truss elements, the axial strain is expressed using the Green–Lagrange strain measure:

$$\varepsilon = \frac{du}{dx} + \frac{1}{2}\left(\frac{dv}{dx}\right)^2 \quad (1)$$

where u and v denote the axial and transverse displacements, respectively.

The nonlinear equilibrium of the structure can be written in residual form as:

$$\mathbf{R}(\mathbf{u}) = \mathbf{F}_{ext} - \mathbf{F}_{int}(\mathbf{u}) = 0 \quad (2)$$

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The solution is obtained using the incremental-iterative Newton–Raphson method:

$$\mathbf{K}_t\Delta\mathbf{u} = \mathbf{R} \quad (3)$$

where $\mathbf{K}_t$ is the tangent stiffness matrix, defined as:

$$\mathbf{K}_t = \mathbf{K}_e + \mathbf{K}_g \quad (4)$$

Here, $\mathbf{K}_e$ is the elastic stiffness matrix and $\mathbf{K}_g$ is the geometric stiffness matrix associated with axial forces. The presence of $\mathbf{K}_g$ leads to stiffness degradation and is responsible for instability phenomena such as snap-through buckling.

2.2. Mixed Finite Element Formulation

To accurately capture the nonlinear response near critical points, a mixed finite element formulation is adopted. In this approach, the nodal displacements **u** and the axial forces **N** are treated as independent variables.

The formulation is derived from the Hellinger–Reissner variational principle. The total potential functional is expressed as:

$$\prod(\mathbf{u},\mathbf{N}) = \int_V \left[\frac{1}{2}\mathbf{N}^T\mathbf{F}\mathbf{N} + \mathbf{N}^T\varepsilon(\mathbf{u})\right]dV - \mathrm{W} \quad (5)$$

where: **F** is the element flexibility matrix,

$\varepsilon(\mathbf{u})$ is the nonlinear strain vector,

W is the external work.

The stationary condition:

$$\delta\Pi = 0 \quad (6)$$

leads to the following system of governing equations:

$$\begin{cases}\mathbf{F}\mathbf{N} + \varepsilon(\mathbf{u}) = 0 \\ \mathrm{G}^T\mathbf{N} = \mathbf{F}_{ext}\end{cases} \quad (7)$$

where G is the compatibility operator relating nodal displacements to element strains.

The incremental form of the mixed system can be written as:

$$\mathbf{K}_t^{(m)} \cdot \Delta x = \mathbf{R}^{(m)} \quad (8)$$

$$\text{with} \quad x = \begin{Bmatrix}\mathrm{u} \\ \mathrm{N}\end{Bmatrix} \quad (9)$$

The mixed tangent stiffness matrix $K_t^{(m)}$ provides improved numerical stability compared to displacement-based formulations, particularly near limit points. As a result, the equilibrium path can be traced accurately without the need for arc-length or displacement-control techniques.

2.3. Modeling of Initial Geometric Imperfections

Geometric imperfections are introduced directly into the element kinematics. Let LLL denote the original member length, and L′Lthe imperfect length:

$$\mathrm{L}' = \mathrm{L} + \delta_0 \quad (10)$$

The initial imperfection induces an initial strain $\varepsilon_0 = \delta_0/\mathrm{L}$

Thus, the total strain becomes: $\varepsilon = \varepsilon(\mathbf{u}) + \varepsilon_0$

In this study, the imperfection δ_0 is modeled as a bounded random variable $\delta_0 \sim U(-\alpha L, \alpha L)$

Where α represents the imperfection amplitude. Typical values considered are:

$$\alpha = 1/2000,\ 1/1000,\ 1/500$$

This formulation enables a direct and efficient representation of fabrication-induced imperfections without additional mesh refinement.

2.4. Structural Optimization Problem

The structural optimization problem is formulated to minimize the total weight of the structure:

$$\min W = \sum_{i=1}^{m} \rho A_i L_i \quad (11)$$

where A_i are the design variables representing cross-sectional areas.

The optimization is subject to the following constraints:

(1) Stress constraint

$$\sigma_i = \frac{N_i}{A_i} \le \sigma_{allow} \quad (12)$$

(2) Displacement constraint

$$u_j \le u_{allow} \quad (13)$$

(3) Stability constraint

The structure must remain stable up to the design load level, expressed in terms of the critical load factor:

$$\lambda_{cr} \ge 1.0 \quad (14)$$

In the numerical implementation, instability is detected when: $\det\left(\mathbf{K}_t^{(m)}\right) \le 0$

or when the nonlinear solution fails to converge.

To handle constraints, a penalty-based objective function is adopted:

$$F = \mathrm{W} + K\sum \max(0, g_i) \quad (15)$$

where K is a large penalty coefficient and g_i are constraint violation functions.

3. Numerical Investigation and Discussion

3.1. Benchmark structure and analysis cases

To evaluate the influence of geometric imperfections on structural stability and optimal design, a 24-bar star dome is adopted as the benchmark structure in this study. The selected benchmark is widely used in structural optimization and nonlinear stability research due to its pronounced geometric nonlinearity and imperfection-sensitive behavior.

The dome consists of 24 truss members arranged symmetrically in a radial configuration, with a concentrated vertical load applied at the apex node. The structural members are grouped into three design categories according to geometric symmetry to reduce the dimensionality of the optimization problem. The material is assumed to be linearly elastic steel with Young’s modulus $\mathrm{E} = 20.340\ \mathrm{kN/cm^2}$

For optimization purposes, the structural members are classified into three design groups according to their geometric location and structural function, namely: apex members, inner ring members, and outer ring members**.**

A concentrated vertical load is applied downward at the apex node, while all outer boundary nodes are fully restrained. Three analysis scenarios are considered in the present study:

1. Perfect Nonlinear Analysis (PNA): Geometrically nonlinear analysis assuming ideal geometry;

2. Imperfect Nonlinear Analysis (INA): Geometrically nonlinear analysis including initial geometric imperfections.

The geometric imperfections are introduced in the form of initial member length deviations, with amplitudes ranging from L/2000 to L/500. As illustrated in Figure 1, the benchmark structure consists of 24 truss members arranged in a radially symmetric star dome configuration.

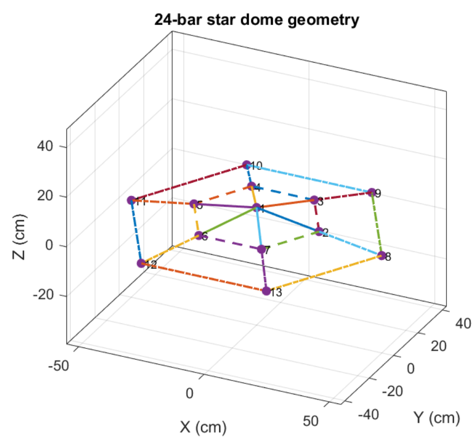

Figure 1. Geometry and element grouping of the 24-bar star dome benchmark structure.

3.2. *Influence of geometric imperfections on structural stability*

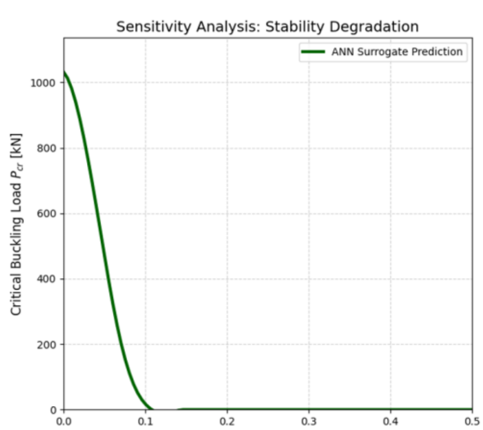

Figure 2. Effect of geometric imperfection amplitude on the critical buckling load of the benchmark shallow dome.

The influence of geometric imperfections on the stability capacity of the benchmark dome is presented in Figure 2. It can be observed that the critical buckling load decreases sharply as the imperfection amplitude increases. For the perfect or nearly perfect configuration, the dome retains a relatively high stability capacity. However, even small geometric imperfections lead to a pronounced reduction in the critical load.

The results clearly indicate the strong imperfection sensitivity of shallow dome structures. As the imperfection amplitude increases beyond a certain threshold, the critical buckling load drops rapidly, indicating a severe degradation of structural stability. This behavior confirms that neglecting initial geometric imperfections may lead to a significant overestimation of the true load-carrying capacity.

3.3. *Comparison of optimal designs under different analysis assumptions*

The optimal design solutions obtained under different analysis assumptions are summarized in Figure 3.

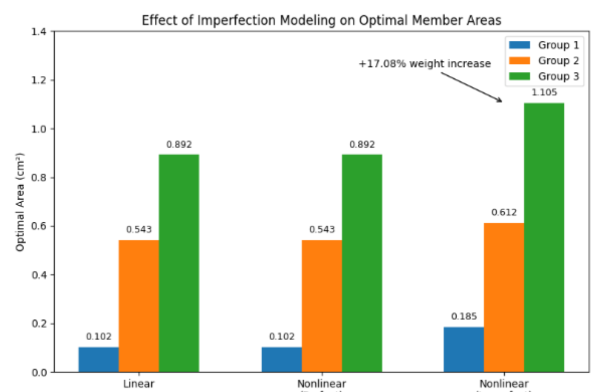

Figure 3. Effect of geometric imperfections on optimal structural member groups.

In contrast, when geometric imperfections are explicitly considered during the optimization process, the optimal design changes significantly. The required member areas increase in all three design groups, leading to a higher structural weight of 0.6247 kg, corresponding to an increase of approximately 17.08 %.

These results demonstrate that neglecting geometric imperfections during optimization may produce unconservative designs that fail to satisfy practical safety requirements.

As shown in Figure 3, the optimal designs obtained from LA and PNA are identical, indicating that geometric nonlinearity alone does not significantly affect the optimization outcome when perfect geometry is assumed. However, when imperfections are included, the optimal member areas increase substantially, leading to a 17 % increase in structural weight.

Figure 3 demonstrates that the outer ring members (Group 3) require the largest increase in area to maintain structural stability when geometric imperfections are included in the analysis.

3.4. Effect of imperfection magnitude on optimal structural weight

The sensitivity of the optimal structural weight to varying magnitudes of geometric imperfections is illustrated in Table 1.

It is observed that the required structural weight increases progressively as the imperfection magnitude becomes larger. For relatively small imperfections of L/2000, the increase in optimal weight is negligible, reaching only 0.6 % compared to the perfect geometry case. This suggests that minor fabrication deviations have a limited influence on the structural design.

However, when the imperfection magnitude increases to L/1000, the optimal weight rises noticeably to 0.6247 kg, corresponding to an increase of approximately 17.1 %. This indicates that geometric imperfections begin to significantly affect the structural stability requirements and must be considered in the design process.

For severe imperfections of L/500, the optimal structural weight increases dramatically to 1.4779 kg, representing a 177.0 % increase relative to the perfect case. Such a substantial increase demonstrates that large geometric imperfections impose severe penalties on structural efficiency and may render the design economically impractical.

These results confirm that maintaining strict fabrication and erection tolerances is essential to ensure both structural safety and economic feasibility in shallow space structure design.

From an engineering design perspective, the present findings indicate that neglecting geometric imperfections may lead to a false perception of structural safety, particularly in lightweight shallow systems where stability governs structural performance.

The study demonstrates that:

- Linear analysis may underestimate instability risk;
- Perfect nonlinear analysis may still fail to capture realistic behavior if imperfections are neglected;
- Geometric imperfections should be explicitly considered during both analysis and optimization stages.

Therefore, incorporating imperfection-sensitive nonlinear analysis into structural design procedures is necessary to ensure safe and economical design of shallow space structures.

Table 1. Sensitivity of the optimal structural weight to varying geometric imperfection magnitudes.

Imperfection magnitude (α)	Optimal structural weight (kg)	Weight increase (%)	Impact assessment
Perfect geometry	0.5336	—	Reference Case
Small (L/2000)	0.5368	0.6 %	Negligible influence
Moderate (L/1000)	0.6247	17.1 %	Significant impact
Severe (L/500)	1.4779	177.0 %	Critical degradation

4. Conclusion

This study investigated the influence of geometric imperfections on the stability behavior and optimal design of shallow space structures through nonlinear finite element analysis and optimization of a 24-bar star dome benchmark.

The results demonstrate that shallow space structures exhibit pronounced imperfection sensitivity, with the critical buckling load decreasing rapidly as the geometric imperfection amplitude increases. Even relatively small imperfections were shown to cause significant degradation in structural stability, highlighting the vulnerability of shallow dome systems to fabrication and erection deviations.

The comparative optimization study further revealed that linear and perfect nonlinear analyses yield identical optimal solutions, both resulting in lightweight but unsafe structural designs when practical imperfections are considered. In contrast, incorporating geometric imperfections into the optimization process leads to safer designs at the expense of increased structural weight.

Parametric investigation showed that the optimal structural weight increases nonlinearly with imperfection magnitude. While minor imperfections (L/2000) have negligible influence on design efficiency, moderate imperfections (L/1000) increase the required weight by approximately 17 %, and severe imperfections (L/500) may lead to weight increases exceeding 170 %.

Overall, the findings confirm that neglecting geometric imperfections may result in substantial overestimation of structural capacity and unsafe optimal solutions. Therefore, imperfection-sensitive nonlinear analysis should be explicitly incorporated into both stability assessment and structural optimization procedures to ensure safe, realistic, and economical design of shallow space structures.

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