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Unified analysis of static and dynamic behavior of steel frames with semi-rigid connections considering geometric nonlinearity

Le Dung Bao Trung[1*], **Nguyen Hai Quang**[2], **Vu Quoc Anh**[1]

[1] Faculty of Civil Engineering, Hanoi Architectural University

[2] Faculty of Mechanical-Automotive and Civil Engineering, Electric Power University

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ABSTRACT

Semi-rigid beam–column connections play a crucial role in governing the overall behavior of steel frames, especially in geometric and dynamic nonlinear problems, where the structural stiffness is no longer constant but depends closely on the working state of the connections. This paper proposes a unified analysis approach for steel frames with semi-rigid connections, in which static and dynamic behavior are considered within a single computational framework, accounting for both connection nonlinearity and geometric nonlinearity. Based on the connection model and a state-updating strategy, the study clarifies the relationship between the effective stiffness of the frame and the nonlinear development of the connections. The results indicate that assuming constant stiffness and natural frequency may lead to significant errors in predicting the dynamic response and fail to capture the nature of resonance shift in the nonlinear domain. The study also confirms the necessity of a unified analysis approach for evaluating resonance and controlling vibrations of steel frames under realistic working conditions.

1. Introduction

In steel frame structures, beam-column connections play a decisive role in the load transfer mechanism, internal force distribution and deformation of the entire system. In practice, these connections rarely work as absolutely rigid or fully hinged connections, but usually have finite rotational stiffness, called semi-rigid connections. The mechanical properties of the connection, especially the moment-rotation relationship, directly affect the overall stiffness, load-bearing capacity and vibration characteristics of the steel frame [1, 2].

Many studies show that considering the stiffness of the beam-column connection can significantly change the horizontal displacement, internal force distribution, as well as the stability and vibration state of the steel frame compared to the ideal connection model [3]. This effect is even more pronounced for slender structural systems, where the behavior of the system depends not only on the element but is also strongly influenced by the characteristics of the connection [4].

In traditional analysis methods, beam-column connections are often assumed to be stiff or hinged to simplify the model. However, these assumptions do not fully reflect the working nature of the structure, especially in nonlinear and dynamic problems [1, 5]. Ignoring the stiffness and nonlinearity of the connection can lead to significant errors in predicting the static behavior, dynamic response and vibration characteristics of the system [6, 7]. In the elastic-plastic domain, the stiffness reduction of the connection during loading or vibration becomes even more difficult to describe if the ideal connection assumption is still used [8].

Previous studies have often considered static, stability, cyclic loading and dynamic problems separately. This approach has not clarified the relationship between static behavior, nonlinear development of connections and dynamic response of structural systems [2].

In reality, the overall stiffness of a steel frame with semi-rigid connections is not a constant but depends on the working state and the degree of nonlinearity. Therefore, vibrational characteristics such as natural frequencies and vibration modes also vary with the loading process. This necessitates a unified analytical approach in which static, nonlinear, and dynamic behavior are considered within the same framework.

This paper proposes a unified analytical method for steel frames with semi-rigid beam-column connections, in which static behavior, stability, cyclic loading, and dynamic behavior is considered comprehensively. The study focuses on planar steel frames with connection models ranging from linear to non-linear elastic-plastic, while also considering the influence of geometric non-linearity. The research results contribute to clarifying the phenomenon of resonance shift and provide a basis for evaluating the dynamic response of steel structures in practice.

In recent studies, the authors developed and validated finite element models for steel frames with semi-rigid connections. Specifically, the equivalent element stiffness model and nodal load vector were verified through comparison with SAP2000 software and

*Corresponding author: Trungldb@hau.edu.vn
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standard problems [9]. In addition, the dynamic model considering the consistent mass matrix, geometric nonlinearity and connection nonlinearity was also validated through standard examples and results published in the literature [10, 11].

Based on that, the current study focuses on developing a unified analytical approach to clarify the relationship between static, nonlinear, and dynamic behavior of semi-rigid steel frames.

The distinguishing feature of this study is the development of a unified analytical framework that allows for the simultaneous consideration of static, nonlinear, and dynamic behavior, where the stiffness of the system is viewed as a state-dependent quantity, thereby directly explaining the variation of oscillation frequency and the phenomenon of resonant displacement.

2. Basic modeling of steel frames with nonlinear semi-rigid connections

2.1. Moment-rotation model of a nonlinear semi-rigid connection

The concept and classification of semi-rigid connections have been presented in [1]. Many moment-rotation models have been proposed, including segmented linear, multilinear, and smooth nonlinear forms based on experimental data. In cyclic loading and dynamics problems, segmented elastic-plastic models are often preferred due to ensuring a balance between accuracy and computational efficiency [12].

In this study, semi-rigid connections are modeled according to the ideal elastic-plastic moment-rotation relationship. This approach allows for the direct description of the stiffness reduction process of the connection according to the working state, thereby clearly reflecting the influence of the connection on the overall behavior and vibration characteristics of the steel frame. Although the actual relationship of the connection is smooth nonlinear, the elastic-plastic model is still capable of representing the transition from elastic to nonlinear state and is widely used in dynamic analysis [2].

Based on that, the research focuses on the variation of tangential stiffness with state, considering it as the quantity that governs the change in vibration frequency and the phenomenon of resonant displacement of the structural system.

2.2. Influence of semi-rigid connections on the overall stiffness of the frame

The existence of semi-rigid connections makes the overall stiffness of the steel frame not only dependent on the geometric and material characteristics of the beams and columns, but also significantly affected by the rotational stiffness at the connections. When the connection stiffness decreases, the lateral stiffness of the frame also decreases, leading to increased displacement and changes in the distribution of internal forces in the system [2].

In the nonlinear domain, when the connection switches to the plastic yield state, the effective stiffness of the frame gradually decreases with the loading process. This decrease cannot be accurately reflected if the stiffness is assumed to be constant but requires updating the working state of the connection throughout the analysis process. Therefore, the nonlinear elastic-plastic semi-rigid connection model plays an important role in describing the load-bearing capacity, stability state and dynamic response of the steel frame [13].

From a mechanical perspective, the stiffness of the frame is no longer an invariant quantity but becomes a state-dependent quantity. This forms the basis for explaining the variation of vibration characteristics, especially the change in natural frequency and the phenomenon of resonant displacement in nonlinear structural systems.

2.3. The role of geometric nonlinearity in the behavior of slender steel frames

For slender steel frames or those subjected to large loads, geometric nonlinearity P–Delta, including the P–δ and P–Δ effects, significantly influences the overall behavior of the system. As the lateral displacement increases, the axial forces in the elements generate secondary moments, reducing the stiffness and stability of the frame [9].

In frames with semi-rigid nonlinear connections, geometric nonlinearity and connection nonlinearity do not exist independently but interact closely with each other. The decrease in stiffness due to plastic yielding of the connection increases the displacement, thereby increasing the P–Δ effect. Conversely, large displacements further promote the nonlinearity of the connection. This interaction mechanism makes the behavior of the frame strongly dependent on the working state and the degree of nonlinearity of the system [2].

Therefore, it is necessary to consider both geometric nonlinearity and connection nonlinearity in the analysis to accurately describe the behavior of the steel frame under different load types [9].

The element models and mechanical relationships presented in this section have been validated in previous studies through comparison with commercial software and standard problems. Therefore, within the scope of this study, the models used as a basis have been validated, allowing for a focused analysis of the influence of the state dependence of stiffness on the static and dynamic behavior of the structural system.

3. Unified Analysis Method

In addition to static and dynamic analysis capabilities like current methods, the unified analysis method proposed in this study aims to clarify the relationship between the working state of the structure and the vibration characteristics of the system.

The focus of the method is to view the stiffness of the structural system as a state-dependent quantity, rather than assuming it is constant. Based on a semi-rigid connection model using the elastic-plastic moment-rotation relationship and a state update strategy, the method allows interpretation of the variation of vibration frequency and the phenomenon of resonant displacement as a direct consequence of stiffness degradation, rather than just numerical results from dynamic analysis.

An advantage of this approach is the use of the same modeling framework to solve static, cyclic load, and dynamic problems, thereby ensuring mechanical consistency and facilitating computational implementation.

3.1. Setting up a frame element with semi-rigid connections

In the unified analysis method, each beam-column element is modeled with two ends having finite rotational connections, characterized by the moment-rotation relationship. The behavior of the connection is directly integrated into the element, allowing simultaneous reflection of the influence of the element and connection on the stiffness and load-bearing capacity of the system.

The shape function of the beam-column element with semi-rigid connections at both ends is constructed to ensure consistency in setting up characteristic quantities such as stiffness matrix, load vector and mass matrix, and is convenient for implementation in the finite element method [9].

3.2. Characteristic quantities of the element

The elastic stiffness matrix of the element is established on the basis of the energy principle, which simultaneously considers the stiffness of the element and the rotational stiffness of the connection. When the connection enters the nonlinear range, the effective rotational stiffness is updated according to the working state, leading to a change in the stiffness matrix at each calculation step.

To consider geometric nonlinearity, the geometric stiffness matrix is constructed from the current axial force in the element, reflecting the influence of large displacements and the P–Δ effect. Since the axial force depends on the working state of the connection, the interaction between geometric nonlinearity and connection nonlinearity is consistently reflected in the process of determining the effective stiffness of the system [9].

The equivalent nodal load vector is used to convert the distributed loads to the nodes, ensuring consistency with the stiffness state of the element. In dynamic analysis, a consistent mass matrix is applied to more accurately describe the mass distribution and vibration characteristics of the system [14]. All characteristic quantities of the beam-column element with two semi-rigid ends, along with how to set and update them in static and dynamic analysis, are presented in detail in [9-11].

3.3. Structural state update strategy

Because the system's characteristic quantities depend on the working state, the analysis process is performed using an incremental-iterative procedure [9]. At each step, quantities such as displacement, internal forces, and moments at the connections are used to update the rotational stiffness, stiffness matrix, and geometric stiffness matrix.

This approach allows for continuous description of stiffness degradation and nonlinear development of the system, while ensuring consistency between static and dynamic analysis.

3.4. Applicability of the method

The unified analysis method can be applied to many types of problems within the same framework. For static analysis, the method allows evaluating the influence of semi-rigid connections on displacement, internal forces, and the stability state of the steel frame.

In the repeated load problem, the elastic-plastic model of the connection allows describing the formation of hysteresis cycles and the cyclic stiffness decay, thereby evaluating the energy dissipation capacity of the system.

For dynamic analysis, the combination of the consistent mass matrix with the stiffness matrix and the geometric stiffness matrix updated according to the state allows for accurate description of the dynamic response and the variation of vibration characteristics over time. Therefore, phenomena such as changes in natural frequency and resonance shift are explained on a clear mechanical basis [11].

3.5. Dynamic problem analysis process considering initial static load

In this method, the dynamic problem is not considered independently but is built on the working state of the structure under initial static load, in accordance with the actual load-bearing sequence.

First, the static problem is solved using the step-by-step loading method to determine the initial equilibrium state. From this state, quantities such as displacement, internal forces, and tangential stiffness are determined and used as initial conditions for the dynamic problem.

The dynamic problem is solved in the time domain using the numerical integration method, in which the equation of motion, considering both geometric nonlinearity and connection nonlinearity, is solved iteratively at each time step. This process allows for accurate simulation of the dynamic response of the system on the basis of the actual working state [10].

4. Static analysis of steel frames with semi-rigid connections

4.1. Analysis of frames under static loads

As presented, the element model and calculation methods used in this study have been validated in previous works through static and dynamic problems. However, to ensure reliability in the current context, the study still conducts numerical analyses with characteristic cases to clarify the influence of semi-rigid connections on the behavior of steel frames under different load conditions.

The existence of semi-rigid connections, whether modeled linearly or non-linearly, significantly alters the internal force distribution and displacement of the steel frame compared to the ideal connection assumption. This effect is particularly pronounced for slender frames or those subjected to large loads, where the overall stiffness of the system strongly depends on the rotational stiffness of the connection.

The analysis results show that, with the same geometric configuration and load conditions, the change in rotational stiffness at the connection can lead to significant differences in extreme internal forces and peak displacements. This directly affects the assessment of the load-bearing capacity of the frame, especially in cases where the structure works near the limit state [9].

From a mechanical perspective, semi-rigid connections reduce the effective stiffness of the system, leading to a redistribution of internal forces between elements and increasing the structural sensitivity to deformation. Therefore, considering the connection characteristics is necessary to accurately reflect the actual behavior of the steel frame.

4.2. Analysis of the frame under repeated loading

Under the action of repeated loading, the steel frame with semi-rigid elastic-plastic connections clearly exhibits nonlinearity and load history dependence. The hysteresis cycles at the connection reflect the energy dissipation capacity of the system, and at the same time show the decrease in stiffness and load-bearing capacity with the number of loading cycles [9].

As the load increases, the connection changes from an elastic to a nonlinear state with gradually decreasing rotational stiffness. This process usually occurs before the beam or column elements reach the full yield state, thus playing a dominant role in the overall behavior of the frame. The continuous decrease in connection stiffness reduces the effective stiffness of the system, affecting not only the static response but also creating the basis for complex nonlinear mechanisms in the dynamic problem.

In addition, the historical dependence of the connection makes the system's response depend not only on the instantaneous value of the load but also on the previous loading process. This needs to be considered in problems evaluating the fatigue strength and long-term performance of structures.

4.3. Stability analysis of the frame

Stability analysis of a steel frame with semi-rigid connections was performed by gradually increasing the load and solving the equilibrium equation, thereby simulating the load-displacement relationship from the initial state to the critical state and frame instability.

The results show that the stability of the frame depends significantly on the connection characteristics. Compared to a frame with rigid connections, a frame with semi-rigid connections usually has a lower critical load due to the decrease in overall stiffness. The nonlinear development of the connection during loading, especially when the connection enters the plastic yielding phase, increases the displacement and accelerates the instability process of the system [15].

From a mechanical perspective, the decrease in connection stiffness increases the influence of the P–Δ effect, leading to a decrease in the frame's stability capacity. Therefore, considering both connection nonlinearity and geometric nonlinearity simultaneously is necessary to accurately assess the stability state of the structure, especially in cases of near-limit operation.

5. Dynamic analysis of steel frames with semi-rigid connections

5.1. Analysis of frames under dynamic loads

Under the action of dynamic loads such as wind, impact loads, machinery, or ground acceleration, the response of the steel frame depends not only on the excitation characteristics but also on the instantaneous state of the connection. Compared to the ideal connection model, considering semi-rigid elastic-plastic connections allows for a more complete description of the interaction between displacement, internal forces, and the working state of the structure.

Models with state-based stiffness updates show significant differences compared to the assumption of constant stiffness, especially in the case of large oscillations or strong loads. The effective stiffness of the frame is no longer constant but varies with the state of the connection and the degree of geometric nonlinearity. The dynamic analysis method is built on the basis of combining the incremental method with direct iteration, in which the equation of motion is solved by the Newmark method for integration over time [10].

The analysis results show that the hysteresis cycles at the connection tend to stabilize over time. Due to the hysteresis damping mechanism, the oscillation amplitude of the system gradually decreases, and at the same time the equilibrium position of the frame can change from the initial state. In addition, the P–Δ effect becomes more pronounced when the lateral load increases, contributing to the change in the dynamic response of the system [11].

5.2. Natural vibration frequency depends on structural state and varies over time

The variation of effective stiffness makes the natural vibration frequency of the steel frame no longer a constant quantity but dependent on the working state of the structure and can vary over time. Studies on nonlinear vibrations of semi-rigid frames show that the assumption of a constant frequency can lead to errors in evaluating dynamic response and the risk of resonance.

Therefore, in dynamic analysis, the vibration frequency should be considered a state-dependent quantity, directly reflecting the degree of nonlinearity of the structural system. The method for determining the natural frequency considering the P-Delta effect is presented in [13]. Accordingly, the time history is divided into discrete steps; at each step, the stiffness matrix, load vector, and mass matrix are updated according to the current state of the system. From there, the eigenvalue problem is established and solved to determine the corresponding vibration frequency.

This approach allows for tracking the variation of natural frequencies throughout the oscillation process, thereby accurately reflecting the dynamic nature of the nonlinear system.

5.3. Resonance shift phenomenon in steel frames with semi-rigid connections

In steel frames with semi-rigid elastic-plastic connections, resonance no longer occurs at a fixed excitation frequency but can displacement during the oscillation process due to the continuous variation of effective stiffness and natural frequency. This phenomenon, called resonance shift, is of significant importance in evaluating the dynamic safety of structures [16].

Resonance shift can occur even when the initial excitation frequency does not coincide with the natural frequency in the elastic state. When the structure enters the nonlinear region, the decrease in stiffness can cause the natural frequency to approach the excitation frequency, leading to a significant increase in oscillation amplitude. This shows that linear models with the assumption of constant frequency may not be reliable enough in predicting dynamic response [17].

The results in [11] show that this phenomenon becomes evident when considering both geometric nonlinearity and connection nonlinearity simultaneously. In some cases, the system may enter the resonance region during oscillation, but it may also exit the resonance region when operating conditions change. This mechanism clearly reflects the state dependence of the system and needs to be considered in structural analysis and design.

6. Significance of the unified analysis method for semi-rigid steel frames

6.1. Benefits of the unified static-to-dynamic analysis method

The unified static-to-dynamic approach allows for a consistent description of the relationship between effective stiffness, the working state of the connection, and the vibration characteristics of the steel frame, instead of viewing static, nonlinear, and dynamic problems as separate problems.

A prominent advantage of the method is the ability to monitor the change in stiffness throughout the loading and vibration process. Therefore, phenomena such as stiffness degradation, changes in natural frequency, and resonant displacement are explained on the same mechanical basis, instead of being observed only as discrete numerical results.

Furthermore, the consistent use of a single element model and state update strategy across various problem types ensures consistency in analysis and facilitates expansion to more complex problems in structural mechanics.

6.2. Implications for resonance assessment and vibration control of steel structures

A significant contribution of this research is clarifying the nature of resonance in steel frames with semi-rigid elastic-plastic connections. Instead of occurring at a fixed excitation frequency, resonance can shift during vibration due to the continuous variation of effective stiffness and the system's natural frequency.

From a design perspective, this shows that resonance checks based on linear models and initial frequencies may not accurately reflect the vibration risk of the structure. Analytical methods need to consider the possibility of resonance shift in the nonlinear domain, especially for structures subjected to strong dynamic loads or repeated loads.

For vibration control problems, solutions based on the assumption of constant frequency need to be adjusted when applied to frames with semi-rigid connections. Combining control measures with state-updated analytical models enhances the reliability of dynamic response prediction while mitigating adverse structural operating conditions under real-world scenarios.

7. Conclusions

This paper proposes a unified analytical approach to describing the behavior of steel frames with semi-rigid, elastic-plastic beam-column connections, encompassing static, stability, and repeated loading behavior, as well as dynamics and vibration characteristics. The core of the research is to view the stiffness of the structural system as a state-dependent quantity, thereby linking the problems within a single modeling framework.

From a mechanical perspective, the study clarifies the dominant role of semi-rigid connections in the effective stiffness, internal forces, displacements, and dynamic response of the steel frame. The results show that the overall stiffness is not a constant quantity but depends on the working state of the connection and the degree of geometric nonlinearity, leading to time-varying vibration characteristics such as natural frequencies and resonance phenomena.

The unified approach allows for a consistent interpretation of the relationship between static and dynamic behavior, while overcoming the limitations of methods based on the assumption of constant stiffness and frequency. As a result, phenomena such as stiffness reduction, frequency changes, and resonant displacement can be explained on the same mechanical basis.

The research results further confirm the reliability of the finite element model developed and validated in previous works, while expanding the model's applicability in dynamic analysis and resonance assessment of semi-rigid steel frames.

In terms of application, the proposed method provides a basis for static analysis, dynamic analysis, resonance assessment, and vibration control of semi-rigid steel frames under real-world working conditions. Furthermore, this approach can be extended to more complex problems such as spatial frames, as well as other structural systems such as precast reinforced concrete frames and composite structures.

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