DYNAMIC ANALYSIS OF THE PLATE ON MULTI-ELASTIC-LAYER FOUNDATION UNDER MOVING LOAD

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Abstract

The dynamic respones of the plate under moving load is an open problem, it has been researched, developed in many majors of solid mechanics, particularly in transport construction. This article presents a methodology of finite element analysis and a solution of the partial differential equation for the structure of the Kirchhoff plate on the multi-elastic-layer foundation bearing moving load. In which, the multi-elastic-layer foundation is described as the homogeneous Winkler foundation. The moving load with mass oscillates together with the plate. The surveys have been studied in order to show the effects of parameters of foundation and load to the oscillation of the plate.

Key words: *Dynamic of structures, Kirchhoff plate, oscillation, moving load, multi-elastic-layer foundation.*

1. Introduction

In the construction field, particularly in transportation, ready-mix-concrete pavement (rigid pavement) is especially concerned. Rigid pavement can be used for high-level roads such as airport roads, expressways, heavy-load access roads at the terminals, city roads, rail-tracks, border patrol roads, local roads.

When constructing rigid pavement, we can take advantage of available materials that can be produced and exploited such as cement, sand, and stone. Although the initial investment cost is higher than the asphalt concrete pavement (flexible pavement), but with low costs in long-term maintenance, long-life operation, high-resistant under weather conditions in Vietnam, and the most important thing are to self-utilized the available materials, which can be exploited and produced autonomously, are an advantage that stimulates a long-term economic development, and avoids the import of materials that cannot be produced domestically.

The behavior of the rigid pavement, which can be modeled by a Kirchhoff plate on a Winkler foundation with a single parameter as calculated by most current international standards (USA, Russia, ICAO ...) as well as Vietnamese ones (22TCN 223: 1995, Decision 3230 / QD-BGTVT). However, the calculation from these current standards still uses static load and multiplies by a shock coefficient to calculate for dynamic analysis. On the other hand, the stress-limit test is uniformly applied at the midpoint on the long edge of the plate (free edge only links against the drift), which is the vulnerable default position of the plate.

As mentioned above, in order to have a better understanding of the behavior of the plate under the effect of moving load to apply in the construction of rigid pavement, this article uses the finite element analysis to establish and solve the partial differential equations of the oscillation of Kirchhoff plate on the multi-elastic-layer foundation under moving load. In which, the multi-elastic-layer foundation is described as the homogeneous Winkler foundation as calculated from the standards. The moving load with mass oscillates together with the plate.

2. Formulation

2.1 Analytical model

Kirchhoff plate on multi-elastic-layer foundation under moving load.

Plate element under time-dependent moving load as Figure1:

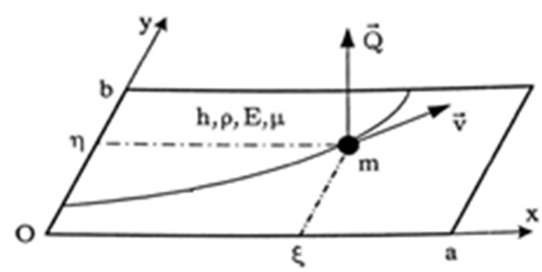


Figure 1. Plate element under moving load.

The plate is considered as Kirchhoff plate based on three basic hypotheses:

- Straight lines normal to the mid-surface remain straight after deformation.
- Straight lines normal to the mid-surface remain normal to the mid-surface after deformation.
- The thickness of the plate does not change during a deformation

The foundation is considered under the elastic phase of a multilayer parameter. Equivalent elastic modulus converted from a multi-layered system to a two-layer system [6]:

$$E_{ch} = \frac{\left[1{,}05-0{,}1\frac{h}{D_{qd}}\left(1-\sqrt[3]{\frac{E_{2,ch}}{E_1}}\right)\right]E_1}{0{,}71\sqrt[3]{\frac{E_{2,ch}}{E_1}}arctg\left(\frac{1{,}35h_{td}}{D_{qd}}\right)+\frac{E_1 2}{E_2\pi}arctg\frac{D_{qd}}{h_{td}}} \quad (1)$$

Where, **E_1** - upper layer elastic modulus; **$E_{2,ch}$** – elastic modulus of lower layer or general elastic modulus of all lower layers; **h** - upper layer thickness; **D_{qd}** - diameter of the converted wheel track, and

$$h_{td} = 2h\sqrt[3]{\frac{E_1}{6E_{2,ch}}} \quad (2)$$

According to N. Gersevanov [6], the subgrade reaction modulus or the elastic modulus of the foundation are characteristic parameters of the strength of the foundation, so they are closely related, when one of the two parameters is known, the remaining parameter can be found:

$$k = \frac{0.65.E_t}{h}\sqrt[3]{\frac{E_t}{E}} \quad (3)$$

Where, **E_t** - elastic modulus of foundation; **E** – elastic modulus of the plate; **h** - plate thickness. Winkler foundation:

$$r_{nen} = r(x,y,t) = k.w(x,y,t) \quad (4)$$

Where, **k** is the equivalent subgrade reaction modulus of a multi-layered system; **w (x, y, t)** is the normal displacement of the foundation at time t.

2.2. Equation of Motion

$$([M_0^e]+[M_P^e])\{\ddot{q}^e\}+([C_0^e]+[C_P^e])\{\dot{q}^e\}+([K_0^e]+[K_P^e])\{q^e\} = \{P^e(t)\} \quad (5)$$

Where, $[M_0^e]$, $[C_0^e]$, $[K_0^e]$ are the mass, damping, and stiffness matrixes of the element under bending.

Where, $[M_P^e]$, $[C_P^e]$, $[K_P^e]$ are the mass, damping, and stiffness matrixes caused by the moving load applied to the element. These matrices contain time-dependent parameters. $\{P^e(t)\}$ is applied load.

Applying Newmark method [7] to calculate the oscillation of the plate, we have the equation of motion in the gradients:

$$([M_0^e]+[M_P^e])\{\Delta\ddot{q}\}_i+([C_0^e]+[C_P^e])\{\Delta\dot{q}\}_i+([K_0^e]+[K_P^e])\{\Delta q\}_i = \{\Delta R\}_i \quad (6)$$

3. The numerical results

3.1 Reliability of the self-coding program

Regarding the reliability of the self-coding program (coded by the Matlab programming language) is tested in comparison with the SAP2000 commercial application: Considering a 4 sided-rigid-connection square plate with the size of 6m x 6m x 0.2m on the Winkler linear elastic foundation with subgrade reaction modulus k. Material characteristics of the plate: E = 3,107kN/m^2; ν = 0.15; ρ = 2500kg/m^3. The plate is subjected to loads that are concentrated in the center of the plate and distributed evenly across the plate (top-down direction). The results of displacement, internal force, and specific oscillation frequency show that the error between the self-coding program with Sap2000 is acceptable. It is possible can say this self-coding program coded by Matlab language is reliable to calculate the structure on the elastic foundation, especially as follows:

- Comparison of displacement at the center of the plate:

Loads	Subgrade reaction k (kN/m3)	Self-coding in Matlab (mm)	Sap2000 (mm)	Obser-vational error
(Point) P=1000 kN	0	99.667	100.266	0.60%
(Uniform) q=100 kN/m^2	0	80.701	81.01	0.38%
(Point) P=1000 kN	10000	27.861	28.104	0.86%
(Uniform) q=100 kN/m^2	10000	11.448	11.45	0.02%

- Comparison of moment M_{xx} at the edge midpoint:

Loads	Subgrade reaction k (kN/m3)	Self-coding in Matlab (mm)	Sap2000 (mm)	Obser-vational error
(Point) P=1000 kN	0	-124.955	-125.203	0.20%
(Uniform) q=100 kN/m^2	0	-183.61	-184.171	0.30%
(Point) P=1000 kN	10000	-4.814	-4.6668	3.15%
(Uniform) q=100 kN/m^2	10000	-46.654	-46.6169	0.08%

- Comparison of the specific oscillation frequency of the plate (where k_n=0)

Mode	Self-coding in Matlab (Rad/s)	Sap2000 (Rad/s)	Observational error
1	63.721	63.592	0.20%
2	129.818	129.460	0.28%
3	129.818	129.460	0.28%
4	190.258	189.049	0.64%
5	232.736	232.009	0.31%

Comparison of specific oscillation frequency of the plate (where k_n=10,000 kN/m^3):

Mode	Self-coding in Matlab (Rad/s)	Sap2000 (Rad/s)	Observational error
1	155.114	155.052	0.04%
2	191.971	191.721	0.13%
3	191.971	191.721	0.13%
4	237.061	236.086	0.41%
5	272.334	271.709	0.23%

3.2 Numerical investigations

3.2.1 Plate on the elastic foundation under dynamic load

- Considering a rigid slab with the size of 3.5 x 4.5(m), 4 sided-rigid-connection, thickness 0.22m, with E= 31.10^6(kN/m^2), Poisson coefficient ν=0.15, specific weight 2.5 ton/m^3;

- Plates placed on a linear elastic foundation with subgrade reaction modulus k_n. Loads, with time dependent, evenly distributed on the plate according to the following mode. With P_0 = 200kN/m2; effective time of load τ = 0.01(s).

- Damping ratio is 0.05 (Rayleigh damping matrix is a linear combination of the mass and stiffness matrix).

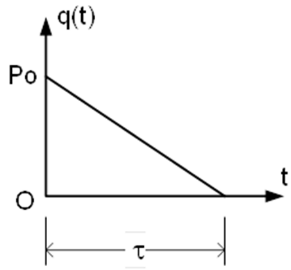
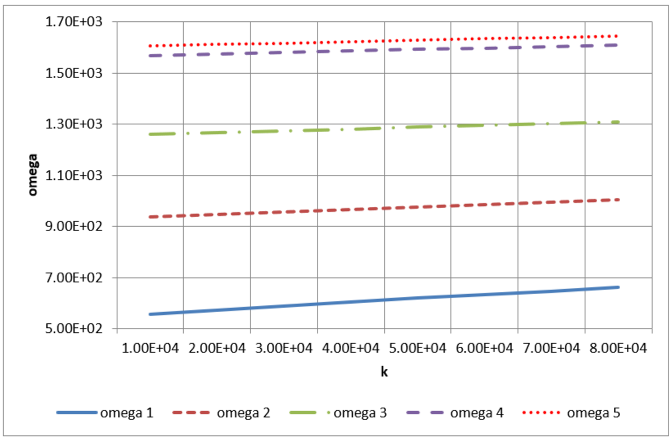


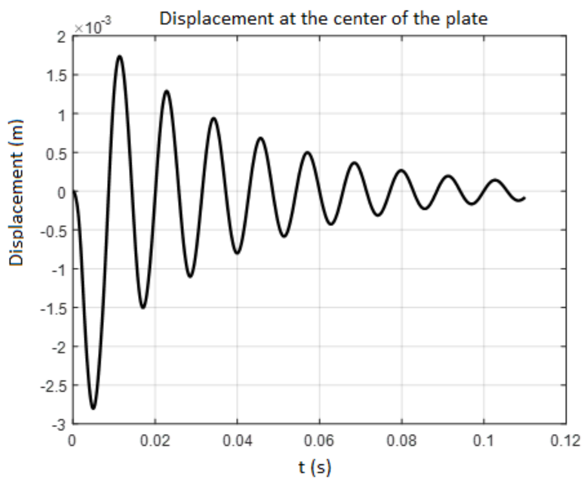
Figure 2: Oscillation mode.

- Specific oscillation frequency and subgrade reaction modulus:

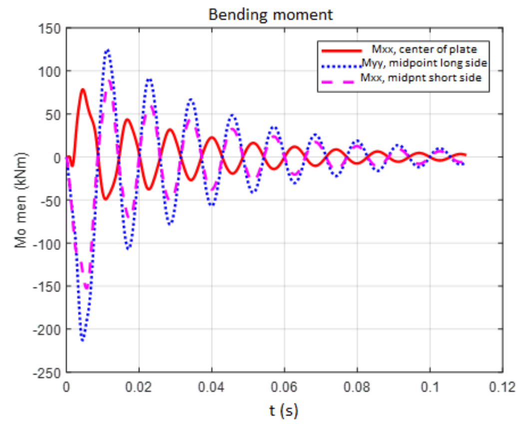


- As shown: when the subgrade reaction modulus increases, the specific oscillation frequency increases accordingly.

- Displacement at the center of the plate, where k_n=10^4(kN/m^3):

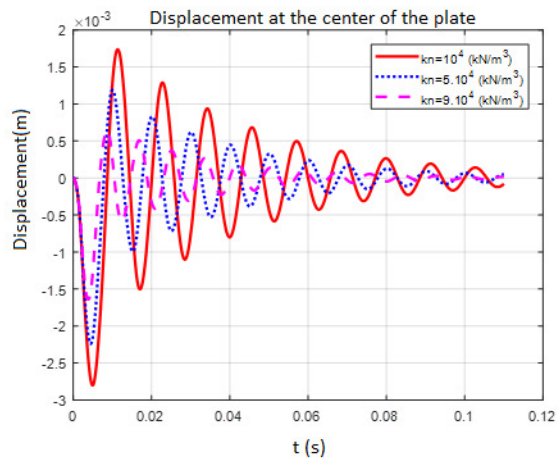


- Bending moment at the center of the plate, where k_n=10^4(kN/m^3)



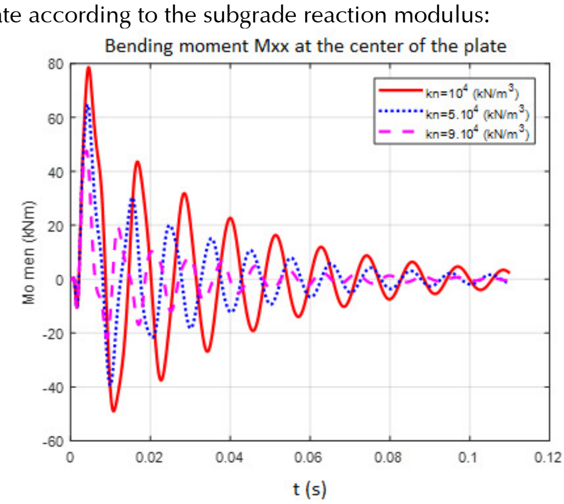
- As shown: the displacement and bending moment on the place decrease with time.

- Investigation of the oscillating displacement of the plate according to the subgrade reaction modulus:



- As shown: when the subgrade reaction modulus increases, the amplitude of displacement of the plate decreases, and the oscillation of the plate turns off faster.

- Investigation of the oscillating bending moment of the plate according to the subgrade reaction modulus:



- As shown: when the subgrade reaction modulus increases, the amplitude of bending moment of the plate decreases, and the bending moment of the plate turns off faster.

3.2.2 Plates on a multi-elastic-layer foundation have a laterally variable stiffness under dynamic loads:

- Ready mix concrete pavement with joints;

- Standard axle load P = 100kN. Road designed for the heavy truck with axle load Pmax = 180kN;

- The top course:

+ Rigid plate with size of 3.50m x 4.50m x 0.22m.

+ Plate is considered 4-sided-rigid-connection (between each plate, the vertical joints have the anti-drift rebar and horizontal joints have the force-transmission rebar);

+ Calculated elastic modulus E_c = 31.10^6(kN/m^2), Poisson's coefficient V_c = 0,15.

- Foundation: The plate is placed on the foundation with the stiffness changes as follows

+ Foundation section 1:

▪ Aggregate with elastic modulus is 300.10^3(kN/m^2), thickness 0.38m, placed directly on the ground.

+ Foundation section 2:

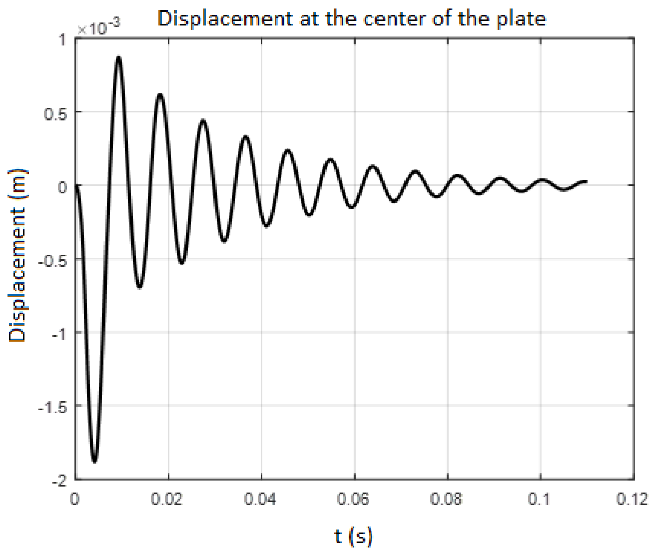
▪ Upper foundation with 5%-cement-reinforced aggregate, thickness 0.20m, elastic modulus at the-age-of-90-days is 1300.10^3(kN/m^2), Poisson's coefficient V_c = 0,20;

▪ The lower foundation with aggregate, thickness 0.18m, elastic modulus 300.10^3(kN / m^2), Poisson's coefficient V_c = 0.35.

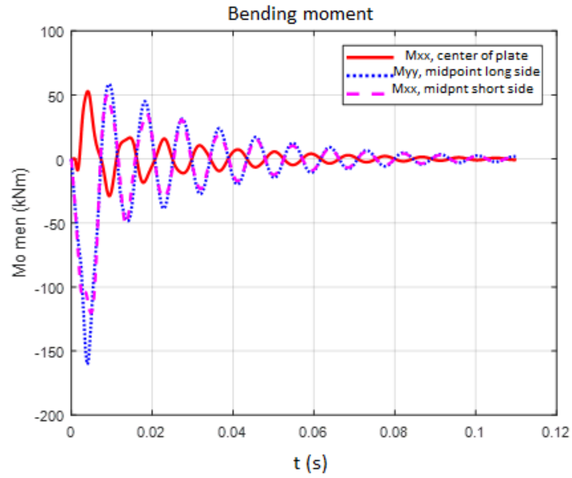
- The ground:

+ Semi-clay with E_0 = 45.10^3(kN/m2).

- Displacement at center of the plate under oscillation:

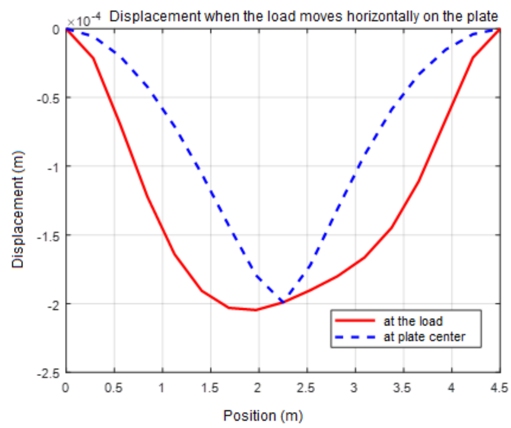


- Bending moment under oscillation:

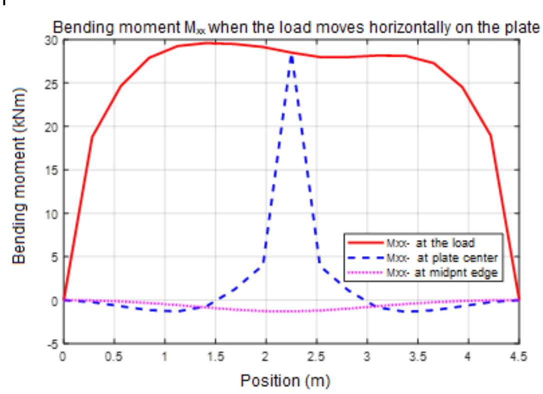


3.2.3 Plates on a multi-elastic-layer foundation have a variable stiffness under moving loads, input data are the same as 3.2.2:

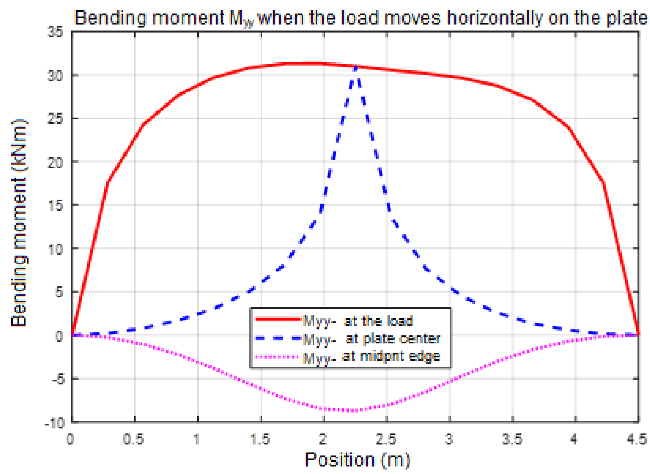
- Displacement when the load moves horizontally on the plate:



- Bending moment M_{xx} when the load moves horizontally on the plate:



- Bending moment M_{yy} when the load moves horizontally on the plate:



- As shown: when the load **moves horizontally**:

+ When the load moves, it will cause displacement

▪ At the position of moving load, the displacement will be bigger when the subgrade reaction modulus is smaller;

▪ At the center of the plate, the displacement will be bigger when the load moves through the position with the lower subgrade reaction modulus.

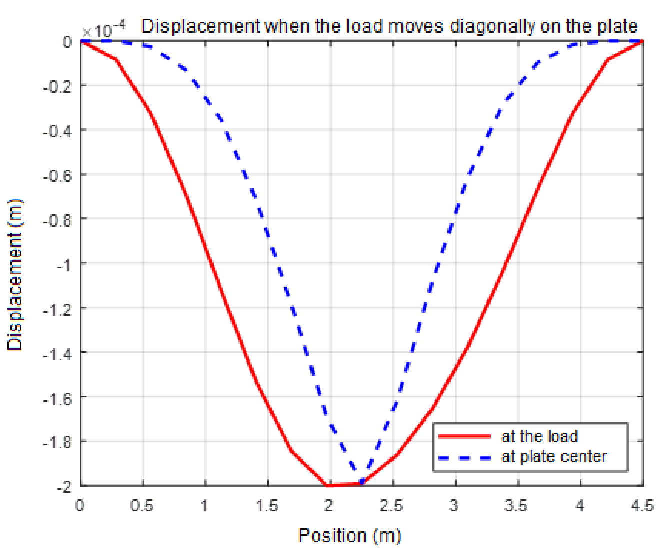
+ When the load moves horizontally, bending moment occurs:

▪ Moment at the load-moving position decreases when the subgrade reaction modulus increases;

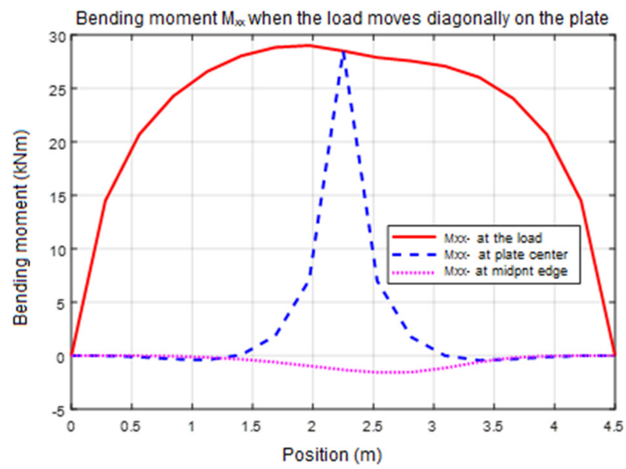
▪ Bending moment at the center of the plate will get maximize at the bottom of the plate when the load passes through the center of the plate.

▪ Bending moment at the midpoint of the long edge will reach its maximum value at the top the plate when the load passes through the center of the plate.

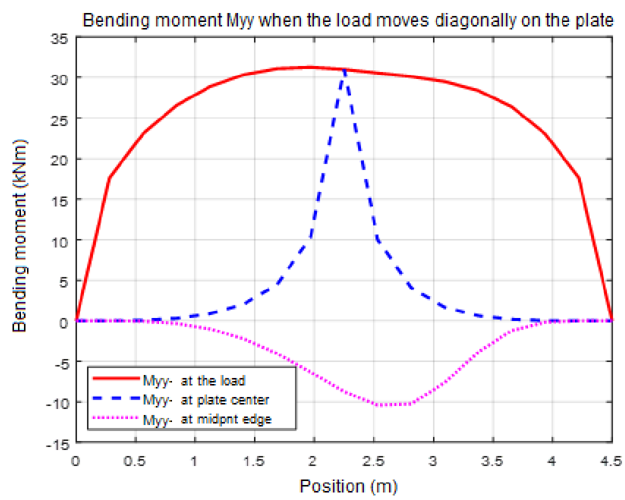
- Displacement when the load moves diagonally on the plate:



- Bending moment M_{xx} when the load moves diagonally on the plate:



- Bending moment Myy when the load moves diagonally on the plate:



- As shown: when the load **moves diagonally**:

+ When the load moves diagonally, it will cause displacement, the same as moving horizontally:

▪ At the position of moving load, the displacement will be bigger when the subgrade reaction modulus is smaller;

▪ At the center of the plate, the displacement will be bigger when the load moves through the position with the lower subgrade reaction modulus.

+ When the load moves diagonally, bending moment occurs:

▪ Moment at the load-moving position decreases when the subgrade reaction modulus increases;

▪ Bending moment at the center of the plate will get maximize at the bottom of the plate when the load passes through the center of the plate.

▪ Bending moment at the midpoint of the long edge will reach its maximum value at the top of the plate when the load reaches the closest at the midpoint of the under-investigating long edge of the plate.

3.2.4 The results of numerical investigation:

- The results of calculation and investigation show that the parameters of the foundation model such as elastic modulus, stiffness, single-elastic-layer or multi-elastic-layer foundation, homogeneous or heterogeneous (stiffness changes horizontally) significantly affect the internal force and displacement of the plate.

- The practical model of the plate on the elastic foundation is quite complicated with the layering and heterogeneity of the foundation. Therefore, taking these factors into account in the calculation of the structure of the plate on the elastic foundation is a necessary and meaningful consideration.

4. Conclusion

- In the calculation results, the displacement and internal force values, caused by the load exerted on the plate placed on the multi-elastic-layer foundation with variable stiffness, were determined. Thereby serving as the basis for the design, audit in construction for rigid pavement and road.

- In addition, the displacement and internal force values determined at the boundary between two zones with different subgrade reaction modulus also provide information for the design of rigid pavement at locations with variable stiffness such as abutments, both sides of underpass or box culvert ... in order to avoid irregular settlements, resulting in cracks, fracture of the rigid pavement, causing inconvenience in the traffic movement of vehicles.

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